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OPERATIONAL EVALUATION OF AIRPORT RUNWAY DESIGN AND CAPACITY

(A Study of Methods and Techniques)

REPORT NO. 7601-6
Contract FAA/BRD-136
January 1963

Prepared for
Federal Aviation Agency
Systems Research Development Service
Research Division
Project No. 412-7-1R

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AIRBORNE INSTRUMENTS LABORATORY
A DIVISION OF CUTLER-HAMMER, INC.
Deer Park, Long Island, New York

OPERATIONAL EVALUATION OF AIRPORT RUNWAY DESIGN AND CAPACITY

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By

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FOREWORD

This report is one of a series of three volumes containing the results of a study program on airport runway and terminal design. The program is a continuation of previous work published under the title, "Airport Runway and Taxiway Design". The three volumes describing the new work consist of this volume, a handbook entitled "Airport Capacity," and a previous volume entitled "Airport Terminal Plan Study." Additional practical applications of the techniques described in this report can also be found in "Airport Facilities for General Aviation," prepared by Airborne Instruments Laboratory under Contract FAA/BRD-403.

ABSTRACT

This report describes a continuation of research into the application of mathematical techniques to the evaluation of practical airport capacity and delays. Since the primary task was to develop a handbook for determining airport capacity and delays by the engineer in the field, the main effort was concentrated on developing existing mathematical models for universal application. Therefore, this report contains the background material relevant to the handbook, describes the mathematical models used, and discusses the preparation of their respective inputs. These inputs vary with runway configuration, runway use, aircraft population, and operating rules (VFR or IFR). The airport surveys that were analyzed to provide input values and operating parameters are also described. An IBM 7090 Fortran program was written to automatically compute the inputs and model outputs in the form of delays versus operating rates and capacities of airport runway configurations. The use and application of this program is described.

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GLOSSARY

The average value (first moment) is indicated by a small letter with subscript. For example, a_1 is the first moment of A, and a_2 is the second moment.

A	Average minimum inter-arrival spacing
B	Minimum arrival service time, B = R + C
C	Average landing commitment interval
CL	Commitment to land point
CT	Cleared to takeoff
D	Inter-departure time for departures
F	Average minimum time required to release departure for takeoff in front of an incoming arrival
FIM	First-come, first-served model
FR	CT(n-1) + J(n)
G	Gap in inter-arrival spacing, G = L - B
H	Interval that starts at end of K
IFR	Instrument flight rules
J	J = H + K
K	Interval that starts when n-1 departure takes off
L	Inter-arrival time for arrivals
λ_L	Arrival rate in landings per hour
λ_S	Arrival rate plus departure rate
λ_T	Departure rate in takeoffs per hour
OR	Off runway
OT	Over threshold
PAM	Pre-emptive Poisson arrivals model
R	Average runway occupancy for arrivals from "over threshold" to "off runway"

I. INTRODUCTION

A. GENERAL

A comprehensive mathematical analysis of airport runway and taxiway design has been carried out by Airborne Instruments Laboratory (AIL) under the direction of the Research Division, Systems Research and Development Service, Federal Aviation Agency. This work has been reported upon previously (reference 1) at a time when the formulation of three basic mathematical models was completed. Since that time, the effort has been devoted to the creation of information that would be of more direct use to the airport engineer in the field. This has necessitated a very close study of IFR operations and various airport runway configurations.

The results of this current effort are incorporated in three volumes: the present volume, an Airport Terminal Plan Study, and an Airport Capacity Handbook. The separate volumes are intended to simplify the use of the information that has resulted from the rather diverse efforts that have gone into the project. It is the purpose of this volume to sum up the theoretical work in a form that will be of interest to those working in the fields of research and development. The mathematical formulations, theoretical and practical investigations, and certain peripheral studies that have been included under the same contractual effort also will be treated. This volume explains and supports the Airport Capacity Handbook, which is concerned completely with the application of the models. The Airport Terminal Plan Study (reference 2), which was prepared by Porter and O'Brien in cooperation with AIL, covers the subject of the terminal building and its supporting systems such as baggage handling and fuel-

ing. The mathematical approach to the probability of gate occupancy (used in reference 2) is presented in this volume as an Appendix.

B. ADVANCED MODEL APPLICATIONS

Since the publication of the last report, the work on the runway mathematical models has continued with several objectives. To provide a comprehensive handbook for determining airport capacity, it was necessary to extend the work previously reported upon into several applications which are more complex than those previously investigated. The first of these were intersecting runways with mixed landings and takeoffs on all runways in VFR. Such configurations consist of two distinct types: (1) intersection occurring within the lengths of each runway, and (2) intersections beyond the runway lengths (open V with operations toward the apex). Second, we had to perform a complete analysis of IFR operations, including the following: (1) single runways with either mixed operations, landings only, or takeoffs only; (2) intersecting and open V runways as in item 1; and (3) additional analysis of close parallel runways where the separation between runways is less than 5000 feet. Techniques for handling all of these situations have now been developed.

In accomplishing this new work, several simplifications and refinements in the basic models were found to be possible. In addition, the entire symbology, which had proven somewhat confusing in the earlier work, was simplified and clarified. The result of all of these improvements has made the previous volume (reference 1) obsolete in many respects. Therefore, a complete explanation of the models, their development, and application will be presented in this report. It should be stressed that this report provides the mathematical and practical groundwork on which the Airport Capacity Hand-

book was based. Therefore, many of the actual conclusions reached during this work will appear in the Airport Capacity Handbook.

In addition to the basic work of developing the models so that they would meet the Handbook requirements, several other peripheral studies were performed and these are reported upon in this volume as Appendices A through E. They include:

Time-Dependent Non-Stationary Runway Model.

Determination of Delay Using Steady-State Models in Non-Stationary Situations.

Effects of Airport Altitude on Runway Capacity.

Analysis of Aircraft Speeds on Approach.

Mathematical Description of Multi-Server Queuing Model Used for Computation of Gate Delay.

Runway/Taxiway Crossings.

II. REFINEMENTS OF STEADY-STATE MATHEMATICAL MODELS FOR VFR OPERATIONS

Three mathematical models were described in the last report.

1. First-Come, First-Served Model (FIM),
2. Pre-emptive Spaced Arrivals Model (SAM),
3. Pre-emptive Poisson Arrivals Model (PAM).

The work previously described established that with suitable inputs the three mathematical models provided a basis for evaluating aircraft delay versus operating rate for single runways and runway/taxiway crossings.

In this new work it was desired to extend this type of analysis (that is, the practical application of mathematical techniques) to the following situations:

1. Intersecting runways in VFR,
2. Dual arrival feed in VFR to multiple runways,
3. IFR operations for all runway configurations.

This section is intended to give the reader a non-mathematical description of the work that was carried out to meet these objectives. Therefore, it is presented in the form of a historical narrative since this method best describes the logic that was used to solve the problems.

It was first established that the original SAM was the most effective model, but that certain rules of procedure concerning the formation of the inputs for single-runway operations could be simplified. These were:

1. The variability in service times.
2. The landing process and its effect on departure delay.

In analyzing these two aspects of SAM as applied to a single runway, it readily became apparent that the model was also suitable for intersecting runways, and that the effects of a dual arrival feed were quite simple to analyze.

A. VARIABILITY IN SERVICE TIMES

The SAM inputs were originally as follows:

1. Takeoff/takeoff interval (S_2),
2. Takeoff/landing interval (S_{11}),
3. Landing/landing interval (OT/OT),
4. Variability of item 3 expressed as a K (Erlang) factor,
5. Runway occupancy for landings (R_1),
6. Runway commitment interval for landings [C_1 = (OT-OT) - R_1],
7. Landing and takeoff rates (λ_1 and λ_2).

Thus, the only variability from average values of service times accounted for in the model was that for the landing-to-landing interval. However, from the airport observations taken up to that time, it was known that both S_2 and S_{11} were extremely variable. For this reason, it was felt that these variabilities should be accounted for in SAM to make the model truly representative of airport operations. Therefore, SAM was initially modified to include the variability of S_2. This was done by introducing S_{22}, which was the second moment of S_2.

In the previous report the validity of SAM was originally checked by comparing actual delays measured at airports against computed delays (derived at identical movement rates) from SAM. This technique was now applied to the

modified SAM model. The result was that, with variability of S_2 included, the computed delays were now far in excess of the actual measured delays.

After some thought and a re-examination of the airport data taken during the surveys, it became apparent that the variability in service times of the various parameters was somewhat complex in their relationships to the average service times. This is best illustrated by considering a typical airport operation of a single runway used by arrivals and departures.

Two arrivals are approaching to land on the runway, one spaced behind the other. Departures are being held awaiting takeoff clearance since the local controller has decided that there is insufficient time before the first arrival to release any departures.

The first arrival lands and rolls down the runway. At this point the controller estimates that there is a sufficient time interval before the next arrival to release at least one and possibly two departures.

If this time interval, or arrival gap, is somewhat short, the controller will request the first departure to expedite his takeoff. If a second departure is allowed to go after the first, the second departure will also be required to expedite the takeoff.

Thus, where the inter-arrival gaps are short, but sufficiently long to permit departures, the corresponding departure service times can also be expected to be short. In actual operations, there is a very strong relationship between the variability of the inter-arrival gaps and the departure service times.

Additional analysis and testing of the model against actual data revealed that excellent agreement was obtained

between observed delay and computed delay if the average values of the service times for departures were used.

B. ARRIVAL/LANDING PROCESS

The conclusions reached concerning the mean service times were applied to arrivals. In the then existing model, the variability in the landing process was described as the K (Erlang) factor, being a function of the standard deviation of the average minimal arrival separation times at the runway threshold.

It was decided to eliminate this particular input from the model and, at the same time, improve some of the concepts of the original model. These latter changes are best detailed by describing the arrival process as it occurs at an airport in VFR.

The previous work had established that the arrival demand has basically a random (Poisson) distribution--that is, if each arrival is allowed to make its own way to a runway without reference to other aircraft arriving on that runway, then some aircraft could get very close to each other and there is a probability that some collisions would take place, the probability increasing as a function of the arrival rate.

This situation is altered in actual operations and pilots of aircraft arriving at a runway in VFR space themselves in such a way that under normal conditions there will be no risk of a collision. These spacings between successive arrivals can be measured at runways where it can be assured that the interval is an average minimum and not the result of natural gaps in the arrival process.

The average minimum spacings vary according to the types of aircraft involved. Theoretically it could be proved

that these intervals, or their absolute minimums, are a function of two basic parameters:

1. The runway occupancy (R) of the first aircraft.
2. The commitment interval (C) of the second aircraft, defined as the time from when the aircraft is committed to land to when it passes over the runway threshold.

If we call the minimum spacing B,

$$B = R + C.$$

The previous work (which was at that time mainly confined to single-runway operations) presumed that C was the time remaining between the "off runway" of the first aircraft and the "over threshold" of the second aircraft.

Thus,

$$C = B - R.$$

Since R can be measured during airport operations, as can B in its average minimal values, the computation of C is quite straightforward. However, it was discovered that if a runway had excellent turnoffs, thus reducing R to very low values, and if B was fixed, the value of C could be so high that it was difficult to reconcile it with the commitment intervals required by aircraft in operational situations. Therefore, the measurements taken at the various airports during the previous work and added to during this current work were re-examined to establish a constant value of C for each class of aircraft. The final values that were obtained were:

Aircraft Class	Type	Seconds
A	Jet transports	18
B	Piston-turbo-prop transports	9

Aircraft Class	Type	Seconds
C	8000 to 36,000 pounds	6
D	Light twin engine	4
E	All single-engine	0

It was now established that at many runways the average minimal values of the arrival/arrival spacings were often longer than R + C and, in fact, it became apparent that there were inter-arrival gaps even when arrivals were spaced at their average minimal intervals.

However, as far as arrivals are concerned, these gaps are unusable and exist for two reasons:

1. The pilots require a "buffer" or safety margin they can use in case of any misjudgments, especially where a fast aircraft is following a slow aircraft,
2. Where a slow aircraft is following a fast aircraft, the closest the two aircraft can be is on the downwind or base leg. From this point they will become further apart so that at the runway threshold the "unusable gap" will be at the maximum.

Therefore, in our original equation,

$$B = R + C.$$

If this is maintained, by stating that although B is the absolute minimum inter-arrival spacing, there may be a gap (G) in the average minimum spacing (A), the following equation applies:

$$A = B + G.$$

Since A is measurable and B is found from R and C, both of which are known or measurable, the equation can be solved.

If arrivals alone are examined, it is found that FIM with the inputs of a_1 (average value of A), a_2 (second moment of A_1), and arrival rate (λ_L) describes the arrival situation and appears to give delays that correlate with real life.

It is interesting to note the effect of runway occupancy on the arrival situation. At many airports the runways are of such a design that normally,

$$A > B$$

or

$$A > R + C$$

However, in calculating arrival delays and/or capacity on a universal basis, as was required for the Airport Capacity Handbook, it had to be assumed that runways would exist where their design (with respect to turnoff locations) would be such that large average values of R could be expected. This would be expected to affect the inter-arrival spacing so that in these cases the following notation must be used. Where

$$R + C > A,$$

$$A = R + C,$$

or

$$A = B.$$

C. RELATIONSHIP OF ARRIVAL SPACING TO DEPARTURES

The conclusions reached concerning the arrival process have their effect on the departure process, and the effect

varies according to the design or configuration of the runways used for arrivals and departures.

1. SINGLE RUNWAY

The task of the local controller in the tower in handling departures on a single runway where arrivals are present is basically a process of estimating time gaps between landings, and then estimating if the gaps are large enough to clear departures for takeoff. This is basically the SAM principle.

The interesting feature of the controller's task is that, provided that the pilots of the arriving aircraft are content with the spacing they have set up, the controller is not concerned with whether each arrival spacing is a minimum (R + C), an average minimum (A), or larger, where a natural gap exists. As explained previously, there are unusable gaps (that is unusable for arrivals only) and natural gaps. The controller is interested in all gaps regardless of how they occur. On a single runway, even with good turnoffs, the unusable gaps will be quite small and very few departures can be permitted to use them--but there may be a few. Since the controller does not differentiate between the two types of gaps when controlling departures, it can be assumed that the same rules will apply to SAM.

If there are 30 arrivals per hour at an airport and R + C is 60 seconds average for each arrival, runway utilization is 30 x 60 = 1800 seconds. Therefore, in 1 hour there is a further 1800 seconds total gap time, having an exponential distribution, in which departures can be released. Naturally departures will not be able to use the entire 1800 seconds since there will be a probability (depending on the arrival rate) that some of the individual gaps will not be long enough to release departures.

When this new concept of the input data was applied to the original SAM, the departure delays that resulted bore a very close relationship to actual observed delays. This was true of the original SAM testing during the previous work and was to be expected; the significant difference now is that

1. Average service times were used throughout, thus simplifying the generation of inputs.
2. The arrival process was more clearly understood and defined.
3. The model now became more adaptable for cases other than "normal" single runways.

2. INTERSECTING RUNWAYS

It should be emphasized at this point that the development of SAM and its inputs, the model testing, and additional airport surveys were all taking place concurrently. Thus, there was continual feedback in both directions between the model work and the surveys. The surveys, together with the results of the model testing, are reported in greater detail in Section IV.

At the same time that refinements of SAM for single-runway operations were being carried out, a start was made on a separate but related mathematical model for intersecting runways. This proved to be a very complicated and difficult task because there could be up to three runways for such runway configurations, each having its own individual departure and arrival service times, and many additional service times for departures and arrivals relevant to each mixture of runways.

The simplification and re-definition of the inputs to the original SAM made it appear as though the same model could be used for intersecting runways, provided that the inputs were correctly defined, measured, and applied.

In the previous work, a survey was made at Atlanta airport; during this new work, Washington National airport was surveyed. In addition, the use of intersecting runways was studied at Chicago O'Hare and Idlewild airports. Some initial study of these airports indicated that SAM would apply and therefore the work on a new model was stopped.

To explain the effect of intersecting runways as applied to SAM, it is important to establish the inputs required and to define them. This has been partly accomplished so far, but should now be consolidated. To ease the transition from the previous report, the old designations are given in parenthesis.

- T Average minimum interval between successive departures (S_2)
- F Average minimum time required to release a departure for takeoff in front of an incoming arrival (S_{11})
- R Average runway occupancy for arrivals from "over threshold" to "off runway" (R_1)
- C Average landing commitment interval (C_1)
- λ_L Arrival rate in landings per hour (λ_1)
- λ_T Departure rate in takeoffs per hour (λ_2)

Figure 2-1 shows these inputs as applied to a single runway.

Using the same basic inputs, modified for intersecting runways, two factors are involved:

1. Alteration of the average service times because of runway design,
2. Alteration of the average service times because of individual runway use by arrivals, departures, or both (mixed operations).

In Figure 2-1, T is obtained by measuring the interval from "clear to takeoff" (or "start roll") of the first departure of a pair to the "clear to takeoff" (or "start roll") of the second departure, where the second departure was "ready

to takeoff" before the first departure started rolling. Also, since there is only one runway, the probability of a takeoff on this runway being followed by another takeoff on the same runway is obviously 1.0.

Figure 2-2 shows an intersecting runway configuration (departures only) using both runways with an equal number of departures on each. Also, for simplification, it is assumed that all departures are the same type of aircraft.

In the ideal and theoretical case, a departure on runway 1 would always be followed by a departure on runway 2, which in turn would be followed by another on runway 1.

The value of T for 1 followed by 2 is the time required for the departure on 1 from "clear to takeoff" to pass through the intersection of runway 1 with 2. This is assumed to be 30 seconds.

T for a departure on runway 2 followed by a departure on 1 would be from "clear to takeoff" to passing through the intersection of 2 and 1. This is assumed to be 20 seconds since the distance is shorter.

Theoretically, the final T for input to SAM is the weighted mean of T. Since the number of departures on each runway is equal, the probability of 1 followed by 2 is 0.5, and 2 followed by 1 is 0.5. Thus, $t_1 = (30 \times 0.5) + (20 \times 0.5) = 25$ seconds.

However, from observations taken during the airport surveys, it became apparent that, because of the random nature of the departure demand (see also reference 1) there is an equal probability of a departure on one runway being followed by a departure on either the other runway or the same runway. Since the interval for successive departures on the same run-

way is identical to T for a single runway (50 seconds), the actual t_1 input for this runway configuration is now:

$$(30 \times 0.25) + (50 \times 0.25) + (20 \times 0.25) + (50 \times 0.25) = 37.5 \text{ seconds.}$$

This very clearly shows the effect of runway design and departure probability on the SAM inputs.

This type of approach to the formation of the inputs was also applied to F and R and checked against actual field data. For example, in Figure 2-2, if runway 2 was also used by arrivals, F would be a combination of single-runway F (takeoff on 2 followed by arrival on the same runway) and takeoff time on 1 from "clear to takeoff" to the intersection of 1 and 2.

Calculation of R would involve a combination of complete R for arrivals on runway 2 where they would be followed by takeoffs on 2, and a fraction of R being the time from overthreshold on 2 to the intersection of 2 and 1 where arrivals would be followed by takeoffs on runway 1.

This briefly outlines the modifications made to the SAM inputs to solve the intersecting-runway problem. The same technique was applied to solving the Open V configurations where operations are made toward the apex of the V. More detailed explanations of the inputs will follow in Sections IV, V, VI, and VII.

D. DUAL ARRIVAL FEED

At the beginning of this new work it appeared as though it might be necessary to develop a new mathematical model to cope with this type of operation. However, the work which led to a redefinition of the arrival process, plus the intersecting runway problem, led directly to the solution of the dual arrival feed process with no extra model required.

The conclusions reached were also backed up by observations made in the field.

Figure 2-3 A-D shows four basic types of arrival feeds to dual runways which are, respectively:

1. A straightforward situation where there are two completely independent traffic patterns.
2. One basic traffic pattern but some diversion to the second runway occurring at the beginning of the base leg turn before turning on final approach.
3. One basic traffic pattern with some diversions to the second runway but, taking place at a short distance from the runway threshold.
4. Two basic traffic patterns but with runway diversion from each final approach to each runway.

These four patterns are based on observations made at various airports during the field surveys. For the purposes of illustration, parallel runways are shown. Except for Figure 2-3D, the same procedures have also been observed at airports having intersecting runways.

To understand the effects of such patterns on airport capacity one must ask the question--"Why do these different types of patterns exist?"

From the observations taken in the field there are three answers to this question. Diversions from one runway to another are made by the local air-traffic controller:

1. To avoid waveoffs because the first arrival of a pair is taking too long to exit the runway. If the controller suspects that a waveoff may be imminent for this reason, and a second diversionary runway exists, he will ask the pilot of the second aircraft to break off and use the other runway. Such procedures give rise to patterns such as those shown in Figures 2-3C and 2-3D, and occasionally 2-3B.
2. To relieve the work load on himself, during high arrival rates. The controller will

divert aircraft between runways to avoid the tricky estimations of arrival spacing close to the runway threshold. This gives rise to patterns such as those shown in Figure 2-3B.

3. To promote extra gaps between successive arrivals on one runway so that departures may be released on that same runway. Figures 2-3C and 2-3D are good examples of this, and Figure 2-3B may occur occasionally.

Having stated the reasons for these procedures, the question can be asked--What is their effect on airport capacity?

The basic operating rule observed in airport operations (and preserved in the application of the mathematical models) is that arrivals have priority over departures. This results in the rule that arrivals delay each other (FIM) and arrivals delay departures (SAM), but departures do not delay arrivals.

Examine the effect on departures first. Since SAM evidently follows the correct rules of operation, the SAM inputs are of interest. These inputs are:

λ_L	Landing rate
λ_T	Departure rate
T	Departure/departure service time
F	Departure/arrival service time
R	Runway occupancy for arrivals
C	Commitment interval for arrivals

Assume an arrival stream on a runway (1) where some departures are waiting to take off. If some of the arrivals are diverted to another, parallel runway (2) during their approach to runway 1, the effect on departure delay is quite obvious.

The primary effect on the departures will be that the landing rate (λ_L) on that runway (1) will be reduced.

Therefore, more gaps will be available for departures and departure delays will be reduced. There may also be some side effects in that, by diverting some of the arrivals to another runway, the arrival population (mixture of aircraft classes) using runway 1 may be altered. In this case, the average values of F, R, and C will change because these values are directly related to aircraft population.

Thus, provided that the number of arrival diversions and aircraft types involved are known, the effects on departures can be computed quite simply.

Next we must examine the arrival problem. In VFR conditions, pilots space themselves in the traffic pattern. The traffic controller generally plays a secondary role in that he monitors the spacing and ensures that pilots are aware of each other and their respective intentions. The pilots, in settling into a traffic pattern, use their judgment and experience and space themselves according to aircraft speeds and their own personal preferences. The most critical part of the whole arrival process begins as the runway threshold is approached. Therefore, the whole circuit pattern tends to hinge upon this point.

To examine a specific case, Figure 2-3C shows one basic circuit pattern. Assume that the primary landing runway (lower runway) has poor turnoffs and that runway occupancy tends to be high for arrivals. If this were the only runway available for landing it might be necessary for pilots to allow greater spacing between aircraft. Referring to the previous discussion of the arrival process, R + C would now be greater than A, where A is the normal average minimal spacing between successive pairs of aircraft. In this case, the delay to arrivals would be higher or, for the same delay, the number of arrivals would be lower. Thus, arrival capacity would be reduced.

However, the pilots and controllers know that there is a second runway so that, in instances where R is very large (for the first aircraft of a pair), the second aircraft can be diverted to the other runway. Obviously, the average minimal value of A is still the limit because, regardless of runway and commitment interval times, the interval A is as close as the aircraft can comfortably get at the threshold despite the values of R or C.

Assume an arrival population of 100 percent Class B aircraft (piston/turboprop transports over 36,000 pounds gross weight) on a single runway. The average interval A at threshold for this class of aircraft in VFR is 64 seconds at a movement rate of 30 arrivals per hour. However, if runway occupancy is 60 seconds average and the commitment interval for Class B is 9 seconds, R + C = 69 seconds. Since this exceeds 64 seconds, then either a large number of waveoffs would have to be accepted or pilots would have to adjust their spacings to allow for longer time intervals. Since the latter is the more practical and safer course, these increased spacings would limit the capacity of the runway.

If a second parallel or intersecting runway is available for diversions, the average interval between successive arrivals can be again reduced to the average value of 64 seconds. Those combinations of aircraft pairs that result in large values of R and/or C can be broken up by the controller by diverting the second aircraft to the other runway.

Thus, the arrival capacity of any single runway handling this type of traffic is 49 movements per hour provided that the average runway occupancy is 45 seconds or less. (Section VI gives a more complete definition of arrival capacity.) An average occupancy of 60 seconds would decrease capacity to 36 movements per hour. However, if a second runway is

available for diversions, the arrival capacity can be expected to increase up to 49 movements per hour--while remaining basically a single arrival circuit pattern.

Figure 2-3B shows a traffic pattern somewhere between the extremes of Figures 2-3A and 2-3C. In Figure 2-3A, the two circuit patterns are completely independent and can be treated separately. Thus, long runway occupancy could affect either or both runways. In Figure 2-3B, some of the arrivals are diverted at a more extreme range than in Figure 2-3C and the secondary arrival feed constitutes an almost independent operation from the basic arrival pattern. For practical purposes in computing capacity, the independent assumption may be taken.

The only time in VFR that arrival capacity can limit airport capacity is

1. A single runway with poor turnoffs is available only,
2. The arrival population includes a high percentage of Classes A and B aircraft,
3. The number of arrivals is considerably in excess of the number of departures.

Even under these circumstances the arrival capacity is usually not a severe problem and any small increases in capacity can be gained by occasional use of another runway. For this reason, it was considered unnecessary to explore situations such as those in Figure 2-3B in greater detail.

Finally, the question arises as to how much a secondary arrival runway will be used for arrivals when it is available. Some study of the field data indicates that it is very difficult to predict how much of the arrival traffic will be diverted. This is not really surprising since the reasons for diversion will vary from one hour to the next. For example, in heavy arrival peak hours, diversions may be made to ease arrival capacity and controller work load. This

may be followed by a period of lower traffic where few diversions may be necessary. Again, departure peaks may give rise to some arrival diversions to allow departures to take off. Crosswinds, runway lengths, and the angle between intersecting runways will all have their effects.

Subject to these conditions, the airport observations have shown that two basic rules apply:

1. Where one basic arrival traffic pattern is used with occasional diversions to a second runway, the percentage of traffic diverted to the second runway is not normally greater than 30 percent of the total, and values between 10 and 20 percent are normal.

2. If the angle between the primary and secondary runways is 50 degrees or less, diversions may be expected. Angles greater than this involve considerable path stretching and diversions from much greater ranges, which would probably prohibit diversions on any general basis. There are some other considerations not mentioned here and these are outlined in Chapter 3 of the Airport Capacity Handbook.

The effects of arrival diversion on departure delays or capacity also should be mentioned in connection with intersecting runways. This, again, is impossible to state in general terms since it depends upon where the runways intersect with each other and the basic use of runways by arrivals and departures. However, if the runway use is known, SAM (modified for intersecting runways as previously described) does allow solutions.

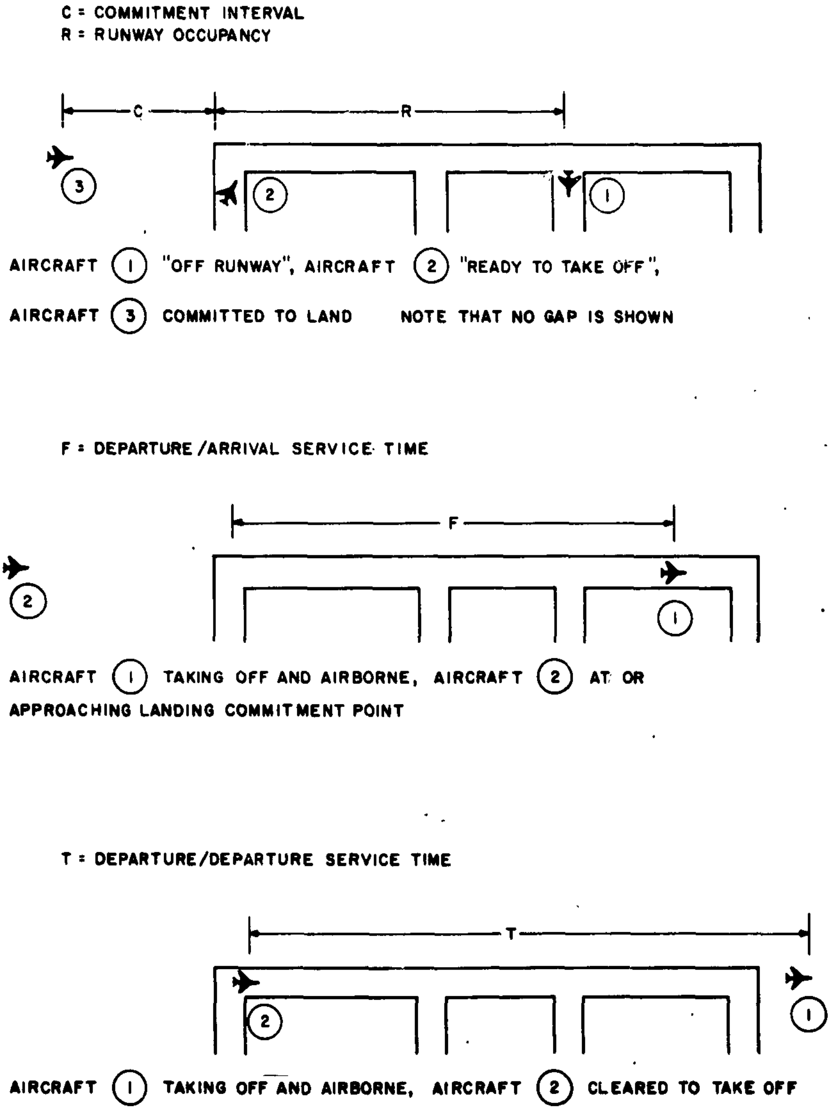
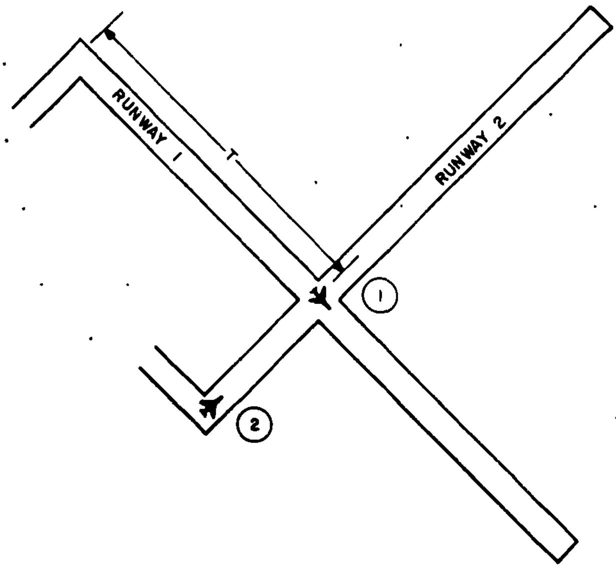


FIGURE 2-1. SPACING FACTORS (INPUTS) FOR PRE-EMPTIVE SPACED ARRIVALS MODEL (SAM) FOR SINGLE RUNWAY

T = DEPARTURE/DEPARTURE SERVICE TIME



AIRCRAFT (1.) TAKING OFF ON RUNWAY 1 AND PASSING THROUGH INTERSECTION OF RUNWAY 1 AND 2   AIRCRAFT (2) READY AND CLEARED TO TAKE OFF

FIGURE 2-2. SPACING BETWEEN SUCCESSIVE DEPARTURES ON TWO INTERSECTING RUNWAYS

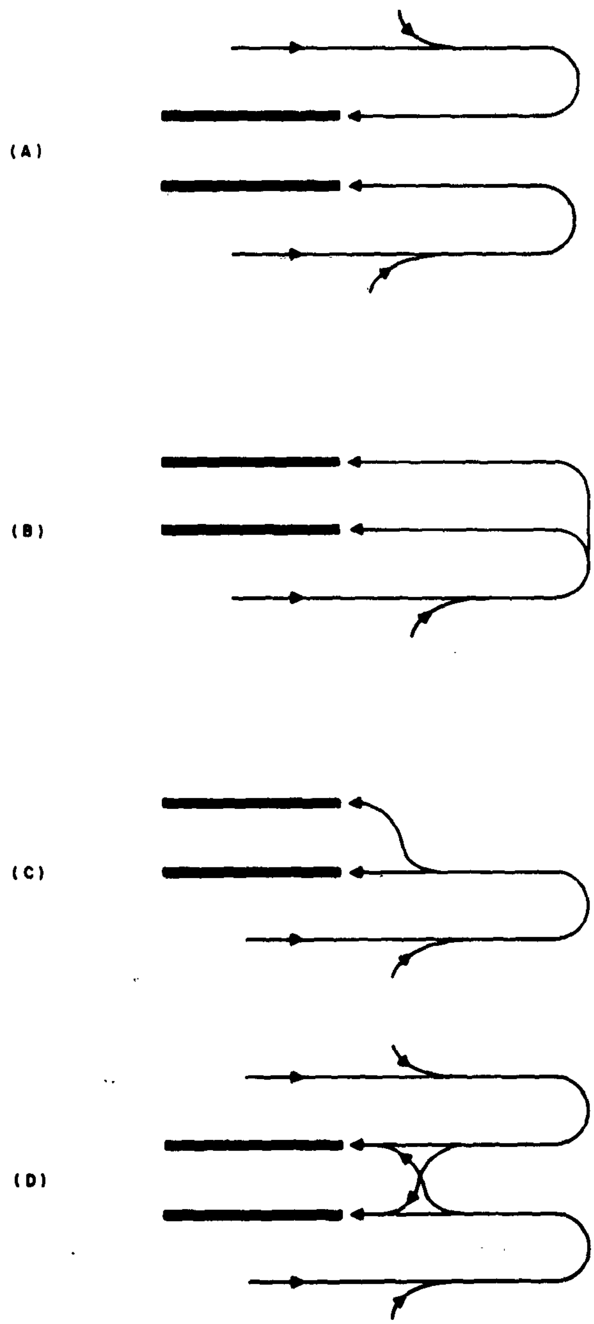


FIGURE 2-3. BASIC ARRIVAL FEEDS TO DUAL RUNWAYS

III. REFINEMENTS OF STEADY-STATE MATHEMATICAL MODELS FOR IFR OPERATIONS

IFR operations were not analyzed in any great detail in the previous work, whereas this new study required a solution of such operations on single, intersecting, and close parallel runways where operations on the two runways are not independent in IFR.

Airport surveys to gather field data required for this new IFR analysis were made at Washington National, Idlewild, Chicago O'Hare, and Los Angeles International airports. These were carried out fairly early in the program together with some additional VFR surveys.

The analysis of the data and adaptation of the mathematical techniques to handle IFR operations was delayed in order to complete the VFR modifications already described. Since these modifications led to a simplification of the model techniques and a greater understanding of the arrival and departure processes, the IFR analysis was greatly eased. This applied both to the single runway and to the intersecting runways. Since the intersecting runway problem had been solved for VFR by modifying the SAM inputs according to runway use, it seemed logical that the close parallel combination could be solved the same way.

However, continuous comparison was made with field data to assure that the field data was correctly interpreted and that assumptions were correct. This also applied to the arrival process since it is more clearly defined in IFR than in VFR, and delays can be measured when aircraft are being stacked.

A large portion of the analysis of the field data was centered on that taken at Washington National and Idlewild airports.

Since the inputs to the models are basically the same in VFR or IFR, the only difference being in absolute values and in some of the processes of formation, the inputs will be listed again and detailed separately.

λ_L	Landing rate per hour
λ_T	Takeoff rate per hour
T	Departure/departure service time
F	Departure/arrival service time
R	Runway occupancy for arrivals
C	Commitment interval for arrivals

A. SINGLE RUNWAY

Since λ_L and λ_T are the demand rates, they are not changed by definition in IFR.

Because R and C are fairly straightforward in composition, they will be dealt with first.

For a given runway design (length, turnoff locations, and turnoff design), the only variations in runway occupancy that can occur for a given type of aircraft will be caused by: (1) variations in final touchdown speed and (2) variations in braking action.

Since wet or slush-covered runways require pilots to use less braking action and because such conditions often occur during IFR--that is, cloud base 1000 feet or less and/or 3 miles or less visibility--occupancy can be expected to change accordingly. Reference 3 indicates that, in IFR conditions, aircraft speeds at or close to touchdown are higher, so that runway occupancy will change.

Both these factors tend to increase runway occupancy times but the increases cannot be graded in a simple fashion. For example, a runway may have turnoffs located so that for VFR conditions they necessitate pilots using limited braking to exit from the runway efficiently. In IFR, the same turnoffs may be perfectly positioned and no increase in runway occupancy will occur. It is possible of course that decreases in occupancy might occur in IFR for some runway/turnoff designs.

The commitment interval (C) can also be expected to change in IFR conditions. The reason for this is that since the point at which this interval starts is where the aircraft is committed to land, low clouds or poor visibility will require that the pilot be assured of a landing somewhat earlier in his final approach than in VFR conditions. Also, the pilot in VFR is usually in a position to decide for himself whether or not he should continue or go around, and since he plays a large part in spacing himself from other arrivals ahead,his judgment can usually be expected to be quite good.

However, in IFR the pilot can be hampered by bad weather and increased workload in flying instruments. Therefore, the burden of determining the "commitment to land" point (CL) falls heavily on the local air traffic controller. Since the controller's experience, reaction time, and radio transmission time (to the pilot) are now involved, an increase in C is inevitable.

During the IFR analysis, various trial computer runs were made using fixed parameters for all the SAM inputs except C, this input being increased by small increments for each successive run. Since T and F are increased substantially in IFR, the value of C can be almost doubled and still remain a small percentage of T and F. For this reason, it was dis-

covered that SAM was not very sensitive to increases in C over that used in VFR. This conclusion was somewhat helpful in the analysis since C is very difficult to measure in IFR conditions so that any estimates made in lieu of actual data would not cause serious errors. Finally, a constant of 10 seconds was added to the VFR values of C for each class of aircraft. Thus,

Aircraft Class	C Seconds
A	28
B	19
C	16
D	14
E	10

Changes in T can also be expected in IFR because, once aircraft are airborne, spacing must be maintained--sometimes over quite long distances from the runway. Analysis of the field data showed that T was dependent upon the population (similar to VFR) and upon the initial departure routes. If only one initial route was available, large values of T could be expected, especially where slow aircraft are followed by fast aircraft. Where a number of departure routes are available, separation is not so critical since aircraft going on different routes are automatically guaranteed separation once the airport boundary is passed. However, there is still the probability that two departures will follow each other on the same route, and this must therefore be included in the formation of T.

The formation of F proved to be relatively simple in IFR. During the airport observations it was apparent that, when an arrival got to 2 miles from touchdown, the controller would not release any departures for takeoff.

However, it was observed that, at any time up to this point, a departure could be released. In fact, it was possible for a departure to receive "clear to takeoff" up to a few seconds before the arrival reached 2 miles inbound. Therefore, the departure could still be on the runway and rolling when the arrival was inside the 2-mile point.

However, both in the model and in real life, the arrival is protected by the commitment interval C so that, at the point where the arrival is committed to land, any previous departures must be well clear of the runway. Therefore, F in IFR for any given pair of aircraft (departure followed by arrival) will be the time from 2 miles to "over-threshold" for the arrival minus its commitment interval.

Interestingly enough, most of the intervals derived in this manner seem to correlate very closely to minimums observed during the IFR surveys, but discrepancies were found where the intervals were between jets, or jets and piston aircraft. In these cases, the times were even less than the VFR minimums for F. However, a 3-mile time computation appeared to give reasonable correlation by increasing the interval, and this was used for the following F service times:

Class	followed by Class
A	A
A	B
A	C
B	A
C	A
D	A
E	A

B. INTERSECTING RUNWAYS

As in VFR (Section II), many of the same considerations will apply here with regard to the SAM inputs. The

primary considerations again are population, runway use, and probability.

The formation of R when modified by intersecting runways is exactly the same as for VFR. Also, C is exactly the same as that used for single-runway operation in IFR.

The primary effects of IFR with intersecting runways are in the formation of F and T.

Figure 3-1A shows a two intersecting runway configuration, where the intersection is close to the runway thresholds. From the field data, it became apparent that the "2-mile rule" of the single runway F still holds for this type of intersecting runway configuration. On a VFR basis, F for the Class B aircraft departing would be the time from "clear to takeoff" to when the aircraft passed through the intersection (typically, 25 seconds). For the same runway configuration and aircraft types, the "2-mile rule" would result in an F of 43 seconds in IFR.

Now consider a runway configuration as in Figure 3-1B. Here the intersection of the two runways is at the far end of the departure runway. The VFR F, using the same definition of F as was used in example A is now 50 seconds. The IFR F, by definition, is still 43 seconds. Thus, a departure would still be at a point 7 seconds before the intersection when an arrival was committed to land. Clearly, this would violate the interval C, so in example B, the IFR F must be the same as the VFR F.

The formation of T in IFR for intersecting runways depends on the use of departure routings. This was very clearly established from the field data.

Figure 3-2A shows two departures using an intersecting runway configuration where two fixes define two initial departure routes.

Where the two aircraft are on different runways and are using different departure fixes, the service time is similar in definition to VFR T--that is, the time for the first departure from "clear to takeoff" through the intersection of the two runways.

If the two departures are using the same departure fix, there can be quite a significant difference in T, since the service time no longer depends upon the runway configuration but on the departure routing. Analysis of the field data indicated that, under these circumstances, T was equivalent to the service time for a single runway in IFR where aircraft are using the same initial departure routing.

C. CLOSE PARALLEL RUNWAYS

To determine the effect of this configuration on SAM inputs, it is again much easier if the actual inputs are examined in detail.

λ_L and λ_T do not change by definition and, since the two runways are close together, the interval C for the arrivals does not change--in other words, this interval must remain protected for all arrivals, whether they are preceded by other arrivals (on the same runway) or by departures on either runway.

The departure/departure service time (T) does not change. Therefore, two close parallel runways are no different than a single runway and the same considerations of departure routings still hold.

Also, since the runways closely approximate the single runway, there is normally no alteration in F. Since the runways are close, the same considerations of waveoff protection to the arrival apply as on a single runway.

There could be an effect on F if the landings are confined to one runway and the takeoffs are confined to the other runway where the two runway thresholds are not coincident. Coincident in this case meaning longitudinally, since there is already some lateral separation. If the takeoff runway in such a case is "ahead" of the landing runway, this would relax the waveoff separation to the extent that the 2 (or 3) mile separation would now be measured from the takeoff runway. If the latter was 1/2-mile ahead of the landing runway, then F would be on the basis of the arrivals being at 1-1/2 (or 2-1/2) miles inbound. This would reduce F in time and increase airport capacity somewhat.

Conversely, if the landing runway were ahead of the takeoff runway, departures would require some increase in F to ensure adequate separation. Again, this extra time is a direct function of the longitudinal separation between the two runway thresholds converted to time for arrivals to cover that distance.

This effect is an assumption since no airports have been surveyed where such a situation exists. However, the modification was required for the analyses leading to the handbook curves, and in the light of our general experience in airport analyses, it is considered that the modifications to F are realistic.

For close parallel runways in IFR, the major difference from a single runway is that the proportion of runway occupancy (R) for arrivals, which causes delay to departures, is quite small.

From observations taken at Los Angeles and Idlewild (runways 4R and 7, open-V configuration where R is equivalent to that of a close parallel configuration), it was apparent that departures were delayed only until the arrival had touched down on the other runway. At this point, the landing is assured

and departures need not be delayed any further. The average time to touchdown from "over-threshold" was calculated for the five classes of aircraft from observed data.

If all the arrivals are on one of the two runways, and departures are confined to the other, maximum benefit accrues from the shorter effective runway time. If some departures also use the arrival runway, then R must be weighted by the probability of an arrival followed by a departure on the same runway. In such a case, R will be lengthened because the R for arrivals delaying departures on the same runway is from "over-threshold" to "off runway."

D. ARRIVAL PROCESS

The priority rule for arrivals in VFR also applies to IFR--that is, in the arrival/departure process, the arrivals have priority over departures, and departures must be released between inter-arrival gaps (SAM). In the arrival process, arrivals may delay other arrivals (FIM).

As described in Section II, the inputs for FIM consist of λ_L, a_1, and a_2. a_1 is the average over-threshold to over-threshold interval, and is measured when the spacing between successive pairs of arrivals is at a minimum.

The essential difference between VFR and IFR is that the intervals between aircraft are governed by the minimum 3-mile spacing required by present regulations. This results in average spacings of greater than 3 miles in terms of time.

As in VFR, there are combinations of average spacings between the various classes of aircraft and the final a_1 is a weighted mean of all the intervals multiplied by their respective probabilities.

As before, the computed delays from FIM were checked against actual delays and very good correlation was found. A great advantage in IFR is that arrival delays are relatively easy to measure if radar photography is available--as it was on this project.

Delays occur mainly in the holding patterns and delay for each aircraft is measured from the time in the stack to the time out of the stack.

A further advantage--from the mathematical aspect--is that arrivals in IFR are confined to the ILS approach and there are very few occasions where aircraft are broken off and diverted to other runways, as happens in VFR.

Further aspects of the arrival process in IFR will be dealt with in Section IV.

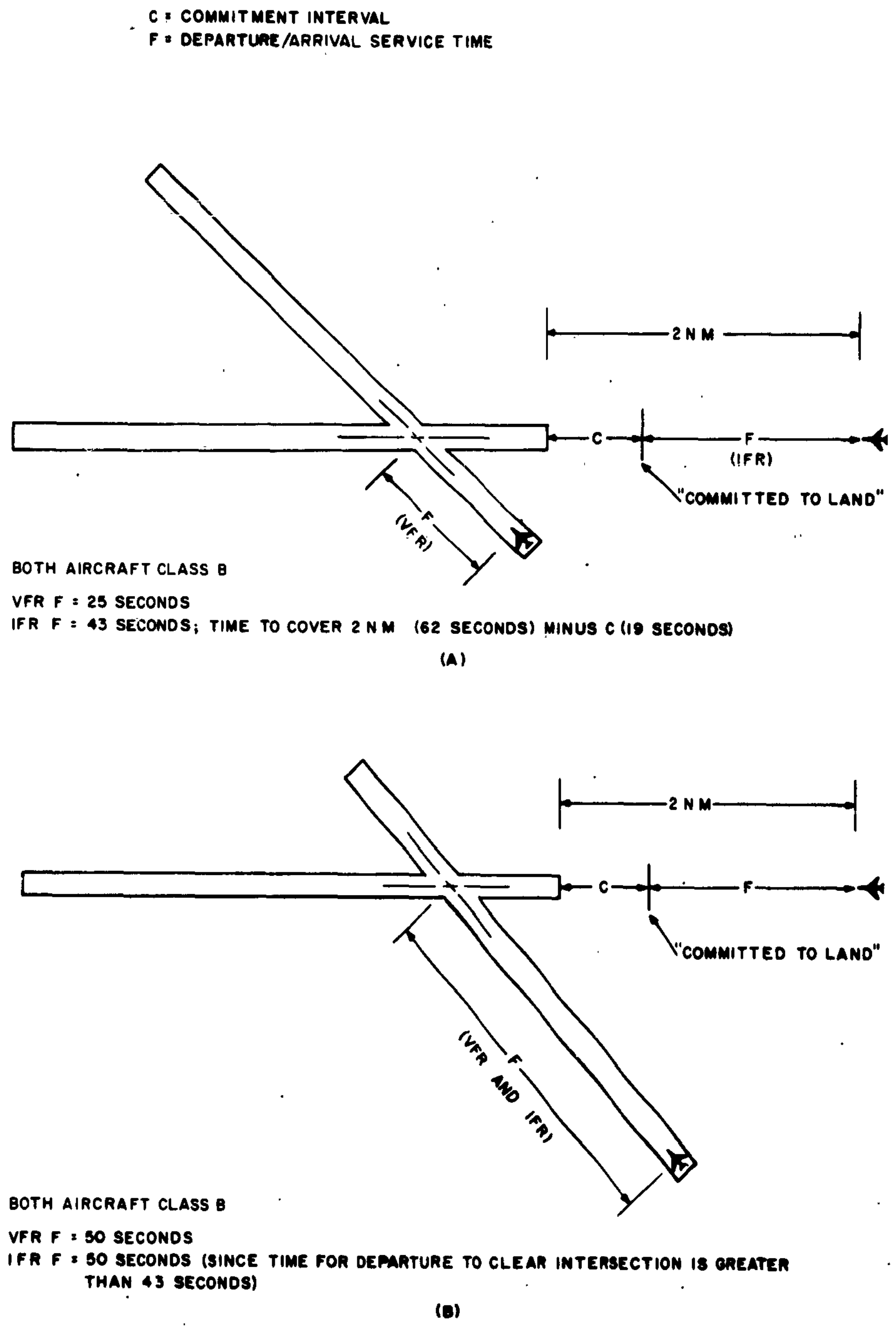
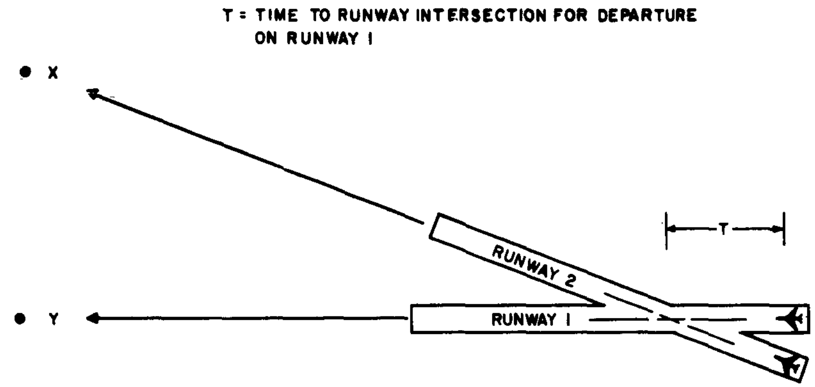
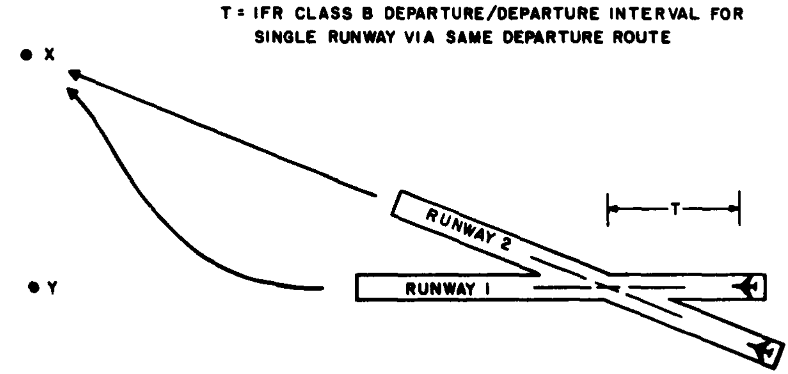


FIGURE 3-1. DEPARTURE/ARRIVAL SERVICE TIME IN IFR (EFFECT OF INTERSECTING RUNWAYS)



CLASS B DEPARTURE ON RUNWAY 1 (THROUGH FIX Y) FOLLOWED BY CLASS B DEPARTURE ON RUNWAY 2 (THROUGH FIX X)

(A)



CLASS B DEPARTURE ON RUNWAY 1 (THROUGH FIX X) FOLLOWED BY CLASS B DEPARTURE ON RUNWAY 2 (THROUGH FIX X)

(B)

FIGURE 3-2. DEPARTURE/DEPARTURE SERVICE TIME IN IFR (EFFECT OF INTERSECTING RUNWAYS AND INITIAL DEPARTURE ROUTE)

IV. AIRPORT SURVEYS AND PERFORMANCE DATA

Sections II and III of this report have given the mathematical background and testing in very general terms. As explained, there was a continual feedback between the mathematical analysis and the field surveys.

This section will cover the field surveys in some detail, give the actual results of the model testing, and list the actual values of the SAM and FIM inputs as measured during the surveys. Some of the tables of values are revisions of those given in the first report, and some are completely new--particularly those applying to IFR operations.

A. METHOD OF DATA TAKING

The actual method of data taking did not materially change from that described in the first report. The following data was taken:

Arrivals

- Aircraft type
- Call sign
- Runway used
- Time "over threshold"
- Time "off runway"
- Runway exit used.

Radar Photography

- Time at outer marker
- Inbound route
- Time in stack
- Time out of stack

arrival delay

Departures

Aircraft type
Call sign
Time enter queue
Time of departure clearance (IFR traffic)
Time "ready to go"
Time move out of queue toward active runway
Runway used
Time enter active runway
Time "clear to takeoff"
Time start roll
Departure route

Using two dual-track (stereo) tape recorders, it was possible to carry out these measurements with three observers. This is quite an advance on previous methods of data collection. The setup used is shown at Figure 4-1. At an airport having intersecting runways, a single runway, or close parallel runways, observer 1 handles departures only, while observer 2 concentrates on arrivals only. At airports where there are wide parallel runways, each observer normally handles arrivals and departures on each runway. Observer 3 is available as a standby--an extra pair of eyes during peak traffic periods--and for changing the radar film and keeping clock synchronization between the cab and the radar camera.

It was found advisable to use our own VHF receivers to remain independent of the tower control personnel by avoiding extra cabling around the cab.

At most airports observer 1 gives departure identification and the following times: enter queue, move, enter active, and start roll. To ease the subsequent task of data analysis, he also logs departure identification and start roll.

This is also useful for check purposes. He also monitors his own transmissions and the local control frequency.

Observer 2 gives arrival identification and the following times: over threshold and off runway. He also identifies the exit used. He maintains an arrival log of identification and over threshold time, and monitors his own transmissions together with the departure clearance delivery frequency.

It was found that, when giving many clock times in quick succession, errors of 5 or 1 minute were quite common and although these were discovered during data analysis and corrected, it was rather time-consuming. Therefore, the latest clock used is a digital hour and minute indicator with a large separate sweep second hand. This clock reduces errors to a minimum.

B. DATA REDUCTION

The graphical technique used for plotting data reported on previously is still being used since it has proved the best method. However, it has been expanded for the IFR analysis. Figure 4-2 shows a sample of a portion of the data taken at Washington National airport in IFR conditions. On the original plot, the departure routes were marked for the takeoffs and different colors were used for each runway. This has been omitted here. Similar plots were made for Atlanta, Idlewild, Chicago O'Hare, and Los Angeles International airports.

The technique for plotting is as follows:

1. Plot time scale and set up queue areas, outer marker points, etc.
2. Plot "over-threshold" and "start roll" times for respective arrivals and departures using the hand-written runway logs.

3. Run through the arrival tape recording and complete arrival runway data--off runway time, exit used, etc.

4. Run through the departure tape recording and plot enter queue, move, enter active, and start roll. Boundary time is an estimate to complete the plot, otherwise it has no significance.

5. Analyze film data and determine stack times and outer marker times. Plot as required.

6. Using time correlation from departure track, determine the following times from the local control frequency recording: ready to go calls (by pilots), and clear to takeoff. Also obtain aircraft call signs and plot as required.

7. Using time correlation from arrival track, determine clearance correct time for all departures from that frequency recording and plot as required.

Any weather information (visibility, wind, etc.) and additional remarks may be added to the plot. The final result is a very complete and easy-to-understand pictorial display of the airport operations. From this plot, all the required spacing intervals can be extracted with relative ease.

The reason for the "departure clearance correct" time being taken is that, during peak traffic hours, many pilots do not call "ready to go" until they have reached the No. 1 position for the runway. Therefore, to ensure that accurate estimates of true ready to go times are obtained, the departure clearance time is required. Also, it does give a clear indication of the delays caused by enroute congestion not caused by the airport runways. This was not specifically called for in this project. It was noted, for example, that delays for departure clearance at Idlewild airport during a survey in February 1961 were very long, while at Washington National in March 1961 departure clearance delays were very short, many aircraft being cleared while still taxiing out from the terminal.

Having described the data taking and method of plotting, the analysis can now be described. Each of the inputs to the SAM and FIM models will be dealt with separately.

C. FORMATION OF INPUTS

Table 4-I gives the five classes of aircraft used for describing aircraft types.

1. T--DEPARTURE FOLLOWED BY DEPARTURE

Definition. The interval between start roll times (or clear to takeoff times) of successive departures measured at the average minimum value.

The minimum value is prescribed when the second departure is ready to go before the first departure starts roll.

VFR

Single runway. Measurements taken during the surveys completed in the previous work were added to and updated with data from the new surveys. Table 4-II presents the latest results. As was described in the previous report, the intervals are subject to the pressure factor (decrease in time intervals caused by increase in airport movements). Therefore, the times are related to λ_S--being the total arrival plus departure hourly rate.

Since completing the surveys for this project, some additional observations have been taken at Chicago O'Hare for the City of Chicago. These observations tend to suggest that the T intervals, where one of the aircraft is Class A, are

somewhat less than those measured during this project. There are three reasons for this:

1. Chicago O'Hare is being operated at capacity at the present time and the pressure factor is very high. Most of the observations taken on this project, where Class A aircraft were present, were at lower movement rates (λ_S maximum, 40) though some λ_S of 50 to 60 were recorded recently at O'Hare.

2. In pairs of successive takeoffs, where the second aircraft is Class A, the reduced run-up time of many jets observed recently may permit closer successive takeoffs.

3. Pilots and controllers are becoming more used to larger numbers of jet aircraft. Thus, spacings are not so restrictive as they were a year or two years ago.

It should be noted that these reductions at high λS only apply to the following aircraft class sequences: A/A, A/B, A/C, A/D+E, B/A, C/A, and D+E/A. The latest Chicago data was checked against the other combinations (B/B, B/C etc.) and no differences were detected in comparison with all previous data.

Intersecting runways, including open V (operations toward the apex). A combination of two time intervals--"clear to takeoff" to start roll, plus "start roll" to desired intersection. The latter is measured from the runway threshold. Table 4-III gives "clear to takeoff" to "start roll" average intervals for the five aircraft classes from the data. It was observed that the pressure factor did have some effect on these intervals, but it was very slight and was ignored for practical purposes.

It should be noted that the figure of 18 seconds for Class A is based on data accumulated up to March 1962. Some very recent observations have indicated that this average may have since become about 9 to 12 seconds. The reason for

this is that pilots are becoming more familiar with the jets and rolling takeoffs are becoming common. Also, the percentage of jets requiring long engine run-up periods for water injection is decreasing rapidly.

The time from "start roll" to a given intersection distance is mainly a function of the aircraft type. Figures 4-3 through 4-7 show time versus distance for Classes A to E.

IFR

Single runway. T is defined in the same way as VFR except that there can be two separate values depending upon whether each pair of departures are on the same initial departure route or proceeding on different routes.

Tables 4-IV and 4-V give the final results as determined from the field data. Table 4-IV gives the intervals where successive aircraft are using the same initial departure route, Table 4-V for different departure routes. It will be noticed that, for some aircraft combinations, the departure intervals in IFR for different departure routes are the same as the VFR intervals.

Intersecting runways. Here the use of departure routes governs the departure spacings in IFR. If two successive departures are using the same initial departure route fix, the interval will be based on that fact regardless of the runway used. This became very clear from the analysis of the Washington data, where all takeoffs from runway 3 used the Riverdale departure fix. Where such takeoffs were followed by a takeoff on runway 36 going via Riverdale, the takeoff intervals were relatively long. Where the takeoffs on runway 36 were routed through the Georgetown fix, the runway 3/36 intervals were the same as in VFR--that is, the

time for a takeoff on runway 3 from "clear to takeoff" to passing through the intersection of runways 3 and 36. Therefore, Table 4-III and Figures 4-3 through 4-7 should be used in such cases.

2. F--DEPARTURE FOLLOWED BY AN ARRIVAL

Definition. The average minimal time required to release and clear a departure in front of an incoming arrival.

VFR

Single runway. F for single runways in VFR is very difficult to measure. At high movement rates, absolute minimums can be observed where on occasion the controller will release a departure very close to an incoming arrival. Since a great deal of field data has been accumulated during this and the previous work it was not too difficult to establish the absolute minimums. It would, however, be desirable to increase the data for these minimums where jet aircraft are involved, but there is enough at the present time to establish reasonable figures.

Measurement of the inter-arrival gaps where departures are ready to go but are not released also provides additional evidence as to minimum F. From this data it is evident that F is subject to the pressure factor.

Measuring the inter-arrival gaps between arrivals where a departure is released provides an indication of F maximum.

With a knowledge of F minimum and F maximum, it was found that the equation

$$F = T - 2C$$

gives a satisfactory solution for F. Also, F is still limited to its minimum values (already known) if the equation gives a solution less than F minimum. Table 4-VI gives minimum values of F for all aircraft class combinations. The equation has been used in all the latest testing of actual versus computed delays and appears to give a satisfactory answer.

Intersecting runways. The calculation of F is quite straightforward here, being identical to T in VFR--that is, the time from "clear to takeoff" to passing through the intersection of the takeoff and landing runway. Table 4-III and Figures 4-3 through 4-7 can be used to calculate F as required.

IFR

Single runway. The 2-mile rule described in Section III applies here. Table 4-VII gives the values of F for each aircraft class combination.

Intersecting runway. Section III gives a complete description. Table 4-III and Figures 4-3 through 4-7, or Table 4-VII, can be used to calculate F as required.

3. R--RUNWAY OCCUPANCY FOR ARRIVALS

Definition.

1. Arrival followed by arrival on the same runway. Runway occupancy from "over threshold" to "off runway" for the first aircraft in every pair of aircraft.
2. Arrival followed by departure on the same runway. Runway occupancy from "over threshold" to "off runway" for the arrival.
3. Arrival followed by departure on intersecting runways other than open V configurations. Runway occupancy from "over threshold" to the intersection of the arrival and departure runway, except in cases where the intersection is toward the far end of the arrival runway. In such cases where the arrivals mostly exit

before the intersection, the field data indicates that the effective runway occupancy ceases as the arrival begins to start to exit the runway. This occurs about 7 to 12 seconds before the actual "off runway" time. In other words, the controller can release a departure before the "off runway" of the arrival. Figures 4-8 through 4-12 show the time from over-threshold versus distance from runway threshold for the five classes of aircraft. For each class, the time also varies as a function of runway length. Therefore, various groups of runway length are shown on each figure. These results are also taken from an analysis of the field data.

4. Arrival followed by departure on intersecting runways where the intersection is beyond the runways--that is, open V configurations on operations toward the apex. Also close parallel runways in IFR. Here the field data indicates that the effective runway occupancy for the arrivals is now time from over threshold to touchdown.

Tables 4-VIII and 4-IX give the values for each aircraft class in VFR and IFR.

The computation of runway time for runway configurations is generally quite straightforward where only a portion of the total runway occupancy is of interest. However, since the very beginning of this airport capacity work, the computation of total runway occupancy--that is, from "over threshold" to "off runway"--has always eluded a simple analysis.

If a runway is in existence and being used, it is a simple (but time-consuming) task to take a number of observations and calculate the average runway occupancy for each aircraft class. However, this is complicated by the fact that the field data indicates that, as the landing rate increases, the average runway occupancy time decreases. Again, this is the effect of the pressure factor. Thus, a simple measurement of runway occupancy time at a λ_L of 10 landings per hour could be expected to be less at a λ_L of 20.

Therefore, it was decided quite early in the beginning of this work that a definition of runway rating should be adopted.

A detailed examination of the field observations was first undertaken to calculate a series of curves that would show the effect of pressure factor. Figure 4-13 shows the family of curves that resulted from this analysis. Use of these curves is best illustrated by an example.

Example. Field observations at an airport show an average runway occupancy of 47 seconds at a λ_L of 15 landings per hour. What will the runway occupancy be at a λ_L of 30 landings per hour? Enter left-hand vertical scale of 47 seconds. Intersection of λ_L 15 occurs on the 45 rating curve. Follow this curve to intersection with λ_L of 30. Now read off the new runway occupancy from the left-hand vertical scale (43 seconds).

Notice that the pressure factor has a greater effect on runways which have poor runway occupancy times than on those with good occupancy times.

All these curves were plotted from a known equation. Thus, if a fixed λ_L of 20 landings per hour is used as a reference line it is possible to specify the runway rating at λ_L 20 as an input to the equation. Therefore, the complete curve for runway occupancy versus λ_L is known. This technique allows two simplifications:

1. It allows a simple definition of runway occupancy by giving a runway rating.
2. It is readily adaptable to a computer program.

However, this only applied to runways where it was possible to measure occupancy by means of a survey. When analyzing airports not yet built, and preparing the airport capacity handbook, it was necessary to predict runway rating.

Also, because it was hoped to keep the handbook presentation as simple as possible, an uncomplicated method of prediction was required.

Some further analysis of the field data led to a solution. It was reasoned that, while individual pilots vary in their landing technique and there are differences between individual aircraft within the five class groupings, it should be possible to determine an exit range for a given class of aircraft. An exit range may be defined as the range of distance along the runway (measured from the threshold) within which the aircraft are in a position, and at such a speed that, if exits are provided within the range, they can be used.

Having presumed that this is a logical assumption, leads to the conclusion that the greater the number of exits within the exit range, the lower the runway occupancy.

Two factors could be expected to alter these assumptions:

1. Effect of airport altitude would increase aircraft true airspeed at touchdown and also lessen effects of propeller or jet braking.
2. Runway length was already known to affect landing performance--the greater the runway length, the less severe is the pilot's braking action.

Because of these factors, the field data was first grouped by runway length and only VFR data from airports whose elevation was less than 1000 feet was considered. Also, only runways having right-angle turns were initially considered.

From the previous work, some estimation of exit ranges by class and runway length was possible. Using these estimations and the actual runway occupancies from the data, we correlated the number of exits available within each exit range with the actual runway occupancies. After several tries,

making small adjustments each time, it was possible to predict runway occupancies of actual runways to within 5 seconds or less on the average.

This left the problem of wet runways, altitude, and high-speed turnoffs. For the latter, the basic technique is similar to right-angle turnoffs except that the exit range is closer to the threshold since aircraft are in a position to use high-speed turnoffs at a speed of 60 mph. In the previous report (reference 1), a considerable amount of work was done on determining the range of this 60-mph point for various classes of aircraft (based on references 3, 4, and 5). This proved invaluable in the calculations. Again, the technique was used to predict runway occupancies at airports where such runway data was available on high-speed turnoffs. These were notably Idlewild and Los Angeles International. In addition, a further analysis was made of runways having angled turnoffs--that is, turnoffs between the right-angled and high-speed types.

Finally there was the aspect of altitude and wet runways (IFR). Since the main effect on the aircraft is to increase the distance along the runway to where the exit range is reached, which is similar to the effect of increasing runway length, it was felt that these parameters could be handled by giving a correction factor to runway length.

For the altitude effect, a study of reference 6 and aircraft performance data yielded the required information on increases in runway length. This allowed a graphical solution of runway correction factor versus airport altitude and aircraft class that was then tested with the runway occupancy data obtained at Denver (elevation, 5331 feet) and reasonable predictions were obtained.

For IFR or wet runways, a correction of 1.1 (10 percent increase) was applied to runway lengths. This was deter-

mined on a trial and error basis using runway data from runways 36 and 4R at Washington and Idlewild, respectively.

The final graphs, tables, and explanation of the technique as applied to actual cases is contained in the Airport Capacity Handbook. Therefore, these aspects will not be covered here.

4. C--COMMITMENT INTERVAL FOR ARRIVALS

Sections II and III of this report describe the analysis leading to the definition of C in VFR and IFR. Therefore, no further discussion is required here.

5. A--ARRIVAL FOLLOWED BY ARRIVAL (FIM MODEL ONLY)

Definition. The interval between successive pairs of arrivals measured at the runway threshold when the spacing is at its average minimum value.

The average minimum value is assured in VFR when:

1. The second arrival is seen to perform any path-stretching maneuvers during downwind, base, or final legs.
2. Two arrivals are both on final approach together in a normal traffic pattern (that is, where arrivals do not come straight in but carry out normal downwind, base, and final legs).
3. A departure is ready to go but not released between two successive arrivals.

The average minimum value is assured in IFR when:

1. The second arrival is seen to be stacked, orbited, or path-stretched before the runway threshold.
2. A departure is ready to go but not released between two successive arrivals.
3. Two or more arrivals are being stacked. Any intervals occurring during such periods of time can be regarded as average minimums, whether or not any of the aircraft making up the intervals have been stacked themselves.

Having specified the conditions for measurement of A intervals, the following comments apply.

VFR

As for most of the model inputs, A is affected by the pressure factor. Thus, there is a decrease of A with increasing λL. The previous report gave a table of values for the various aircraft class combinations which has been updated and revised in this report as Table 4-X.

IFR

The new data in Table 4-XI gives the results of the measurements obtained from the IFR surveys.

Several points are of particular interest here.

Pressure factor does affect the interval A in IFR. From the data gathered so far, the reduction in service times parallels the VFR case. Figure 4-14 is a composite graph showing this effect.

The graph shows A (time) versus λ_S (total movement rate) for Class B followed by Class B. The horizontal hatched lines show the values for A for 3- and 5-mile average spacings between successive aircraft. The dots are individual spacings taken direct from the field surveys under IFR conditions.

The lower curve shows the basic VFR A. When the average values of each set of IFR spacings were plotted, it was evident that a curve paralleling the VFR curve, but greater by 61 seconds, passes through or close to the IFR averages. The top curve is, therefore, the average interval A for Class B followed by Class B in IFR.

Another interesting feature here is that, at the higher movement rates, the average IFR spacing is quite close to the specified 3-mile rule. It will be seen that some intervals apparently fall below the 3-mile line. This should not be construed as necessarily violating the rule in every case, since the 3-mile line is an average based on Class B aircraft average speeds. Some aircraft cover 3 miles somewhat faster than others and, therefore, fall below this 3-mile line.

What it does indicate is that the approach controllers are performing very well with these types of aircraft; the average spacing at the higher movement rates being 3-1/2 miles.

Where jet aircraft (Class A) are concerned, there is a general lack of data at the higher movement rates (above λ_S 25) but the data gathered on this project indicates that average intervals between Class A aircraft are in the order of 6 miles. Recent surveys taken at Chicago O'Hare for the City of Chicago tend to suggest that spacings there may be somewhat less than this for Class A aircraft. This has not been confirmed, but it does indicate that data collection of this sort ought be done fairly regularly at such airports if the model inputs are to be kept up-to-date.

Theoretical analysis. Many analyses of IFR approach feeds and capacities done in the past have assumed that the length of the common path (ILS) will have an effect on capacity since spacings between aircraft having dissimiliar speeds will be necessarily increased on long common paths. This seems to be a valid theoretical assumption.

However, an analysis of the radar photography and time data from the surveys indicate that this apparently does not have a major effect. For example, it would appear at first sight that the length of common path at Los Angeles is twice that of Washington National. At Los Angeles, the air-

space approaching the airport (runways 25L and 25R) is very restricted because of adjacent airports and mountains, and the majority of arriving aircraft are coming from the east. However, by means of radar vectoring and speed control, the approach controllers can bypass fast aircraft around the slow ones; in many cases, the light aircraft are kept clear of the ILS until the last possible movement consistent with safety. Also, at Washington National, aircraft were vectored onto the ILS in such a way that there was, in effect, no common path beyond the outer marker in many cases. The same is true at Idlewild.

There is not enough data yet to absolutely prove that the length of common path does not affect capacity, but the evidence so far suggests that controllers use techniques to avoid the effect.

Another question has arisen many times during the VFR and IFR analyses, both in this and the previous work--do poor controllers have an adverse effect on capacity?

First, it should be stated that when performing a capacity analysis, especially where it leads to an economic analysis of airport design, one cannot plan on anything other than the average controller. Obviously "good" or "bad" controllers could only have short-term effects on airport capacity even if this were true.

The VFR analyses so far have shown that there are few if any such effects. This is partly because of the fact that the pilots are also involved, and the combination of "good controllers" and "bad" piloting is just as likely as both being "good" or vice versa.

From the data taken in IFR, it appears that the same is true, but here the evidence is not as clear as in VFR. There is a suggestion that, if the controllers are not

restrained by local traffic rules (imposed on top of the universal traffic procedures) or by such things as a lack of departure routes, then there may be a learning factor as applied to the jet aircraft. In other words, there is some evidence (as yet unchecked) that high movement rates at some airports in IFR are inducing higher learning rates among those controllers, and spacings are being reduced within the rules--at least those involving jet aircraft. One reason for this is perhaps that the controllers are using speed control intelligently and in fact they are being encouraged to do so.

At one airport, controllers have recently been instructed to use 6-mile minimum spacing on arrivals on one particular runway to alleviate the noise problem. If this rule is followed, reduced capacity and/or increased arrival delays are unavoidable. An added side effect is that the controllers will not have the opportunities to get used to high movement rates and poorer performance is inevitable.

Also, it should be repeated that, in discussing aspects of common path lengths, there is a definite effect on departure capacity as already observed. However, at Idlewild on runway 31L, recent surveys (conducted under Contract FAA/ARDS-605) have indicated that departure capacity on this runway is most severely restricted because there is but one departure route and its length is much longer than observed at any other airport. This does seem to have a serious effect. Another aspect should be mentioned. Chicago O'Hare is presently faced with very high traffic demands coupled with a relatively low capacity airport configuration under certain wind conditions. When the wind is from the west arrivals can use runway 32L (length 11,600 feet) but cannot use 32R because this would result in excessive delays to departures on that runway. Also, the number of arrivals on 32L has to be limited because of the departures on that runway as well.

The ideal situation is to allow some arrivals on 32L and the remainder on runway 27, and departures on 32L and 32R. Until recently the air traffic procedures did not allow independent traffic patterns on 32L and 27. Therefore, this would not have allowed maximum use of these runway for arrivals. However, as a special case, the rules have now been changed for O'Hare to allow this provided that weather conditions are more than 2500 feet of cloud base, and more than 6 miles visibility, together with some coordination between the two runways.

This leads on to the final point. The SAM/FIM model combination has proved its validity in giving answers consistent with airport surveys. If the air traffic control rules change, or operational procedures or practices change, they do not make the model unusable. Such changes only affect the inputs to the models and, provided that the effect of such changes is either measured (in the field) or correctly assumed, then airport capacity and/or delay can be correctly computed.

With this in mind it has become apparent that the technique of field surveys with proven models is a most powerful tool for airport design studies.

D. MODEL TESTING

Table 4-XII gives the final results of the model testing, where the actual observed delays have been compared with computed delays. The computed delays have been calculated using the full IBM 7090 computer program (Section VI) and incorporating the mathematical model and input routines described in Sections II, III, and V.

All these tests are the result of the new surveys conducted during this recent project with the exception of Wichita and Miami, which were completed on the previous contract. For check purposes, these two cases were re-runs using

the full computer program. In the previous report, Wichita had a computed delay of 0.6 minute. The computed delay with the new program is 0.5 minute (actual delay, 0.2 minute).

The Miami test previously gave a computed delay of 4.6 minutes against 2.8 minutes actual delay. This was the worst correlation of all the previous test cases and though statistically valid (from the purely mathematical aspects), it was not a close correlation in the practical sense.

The Miami case was of particular interest because during that particular survey there was some use of intersecting runways though the majority of aircraft used a single runway. During the previous testing, there was some difficulty in running tests on intersecting runways. Therefore, this example was run as though it was a single runway only.

Using the full computer program, which allows automatic computation of inputs for many runway configurations, the Miami case was run as it actually existed during that survey. Table 4-XII shows that the computed delay is now 3.1 minutes compared with 2.8 minutes actual delay.

This case is interesting in that it shows the advantage of intersecting runways over single runways (provided that the runway intersections are favorably located); the single runway resulted in a much higher figure of delay than did the intersecting runway.

The model testing was used to check the validity of application of the finally accepted models, and to refine the definition of model inputs through a recycling procedure. The models were tested against actual operations. Spacing factor inputs were then refined to improve correlation and these refinements incorporated into the composite compilation of spacing factor inputs. The models were then retested using the composite of spacing factors.

An example of the Washington National IFR analysis will illustrate this. After completion of data taking and data reduction for Washington, the initial spacing factors were determined. These factors (from National data only) were used for initial testing of the models. The initial testing indicated better agreement should be obtained, so the operation was re-analyzed to determine that some redefinition of spacing factors was necessary. For example, the effect of departure routings and how to provide for this effect was learned through this process. When reasonable agreement was attained for the Washington test period, IFR operations at other airports were analyzed. Gradually the spacing factors from those airports that were surveyed were summed into a composite curve for each factor expressed in time versus movement rate (to include the pressure factor). The composite curve was programmed as part of the computer program for generation of model inputs. Finally, the Washington National observation period was retested using the computer program of composite spacings. The correlation obtained between computed and actual delay is shown in Table 4-XII. Table 4-XIII repeats the results of the model tests included in the first phase of the contract (reference 1). Not all of these cases were retested under the present phase. Where retesting has been accomplished, it has been done using the broader input data or spacing factors now available.

The model inputs as gradually developed and assembled have become a broad enough sampling to represent a national standard measure of the input values. The model tests appearing in Table 4-XII are those performed at the end of the process of testing and sharpening input data. They are thus indicative of the correlation to be anticipated if one goes to any civil U.S. airport and performs

the necessary field observations, data reduction, and model computations. Should the correlation not be good--say the delay observed is markedly less than the model prediction--one can anticipate that closer study will show an unusual performance is being accomplished which has reduced input values for that case. The comments under previous section C1 on O'Hare observations illustrate this point.

Since the model inputs are based on current measured data, they should be checked periodically by additional field observations, for the spacing factors may change as new procedures are developed and as the operation of new aircraft becomes routine. Broadening of observed data for IFR operations of heavy jet aircraft would be particularly desirable as the data available during the field work of this contract has been more limited than would be desirable. Further, IFR procedures and performance are gradually being improved to increase IFR capacities.

TABLE 4-I

AIRCRAFT BY TYPE AND CLASS

Class	Description	Type
A	All jet aircraft normally requiring runway lengths in excess of 6000 feet for takeoff and/or landing (corrected to sea level).	Boeing 707 and 720 series Douglas DC-8 series. Convair 880 and 990 Sud-Aviation Caravelle DeHavilland Comet BAC VC 10
B	(1) Piston and turboprop aircraft having a normal loaded weight of >36,000 pounds (2) Jet aircraft not included in Class A but having a normal loaded weight >25,000 pounds.	BAC 111 Boeing 727 Lockheed Jetstar Lockheed Electra BAC Vanguard Vickers Viscount Douglas DC-6 and DC-7 series Lockheed Constellation Bristol Brittannia Convair 240, 340, and 440 Martin 202 and 404
C	(1) Piston and turboprop aircraft having a normal loaded weight of >8000 pounds but <36,000 pounds. (2) Jet aircraft having a normal loaded weight of >8000 pounds but <25,000 pounds	Fairchild F-27 Grumman Gulfstream Douglas B-26 Lockheed Lodestar and Learstar series Douglas DC-3 Beech 18 series North American T-39 Potez 840 Aero Jet Commander DeHavilland 125

TABLE 4-I (cont)

Class	Description	Type
D	All light twin-engine piston/turboprop aircraft with <8000 pounds normal loaded weight and some high-performance single-engine light aircraft.	Beech 500 Twin Bonanza Aero Commander Beech Queen Air Beech Travelair Piper Aztec Piper Apache Cessna 310 Cessna Skyknight Beech Bonanza and Debonair DeHavilland Dove
E	All single-engine light aircraft other than those included in Class D	Piper Cub, Tripacer, Pacer, etc. Cessna 140, 150, 170, 180, and 210 series Piper Cherokee Piper Comanche Beech Musketeer DeHavilland Beaver (L-20) Mooney M20 Aeronica Champion

TABLE 4-II

T, AVERAGE MINIMUM SPACING BETWEEN SUCCESSIVE DEPARTURES ON SAME RUNWAY (VFR)

Class A f/b* Class A		Class A f/b* Class B	
λ_s	T (second)	λ_s	T (second)
20	72.0	20	79.0
30	69.8	30	76.8
40	68.0	40	75.0
50	67.0	50	73.8
60	65.8	60	72.8
70	65.0	70	72.0
80	64.2	80	71.2
90	63.5	90	70.5
100	63.0	100	70.0
110+	63.0	110+	70.0

Class A f/b* Class C		Class A f/b* Class D & E	
λ_s	T (second)	λ_s	T (second)
20	83.2	20	85.5
30	74.8	30	78.5
40	69.4	40	73.5
50	65.5	50	70.4
60	62.5	60	67.5
70	59.8	70	65.5
80	57.8	80	64.0
90	55.8	90	64.0
100	55.0	100	64.0
110+	55.0	110+	64.0

Movement rate (λ_s) values are given up to λ_s = 110, but this should not be interpreted as being of any significance other than the fact that it shows the full range over the curve. For example, a runway handling all Class A aircraft would reach capacity well before a λ_s of 110 movements per hour. However, at an airport handling only 1 percent Class A aircraft and a large population of Class D and E aircraft, the capacity could well approach λ_s = 110 movements per hour.

λ_s = movement rate.

T = average minimum spacing between successive departures.

* f/b = followed by.

TABLE 4-II (cont)

Class B f/b* Class A

λ_s	T (second)
20	80.4
30	76.0
40	73.8
50	71.5
60	69.8
70	68.4
80	67.2
90	66.0
100	65.3
110+	65.0

Class B f/b* Class B

λ_s	T (second)
20	81.5
30	71.0
40	64.5
50	60.0
60	56.2
70	53.5
80	51.0
90	49.0
100	48.0
110+	48.0

Class B f/b* Class C

λ_s	T (second)
20	59.0
30	54.6
40	51.5
50	49.2
60	47.5
70	46.0
80	45.0
90	45.0
100	45.0
110+	45.0

Class B f/b* Classes D & E

λ_s	T (second)
20	77.5
30	63.8
40	55.8
50	49.5
60	45.5
70	42.4
80	39.8
90	39.0
100	39.0
110+	39.0

Class C f/b* Class A

λ_s	T (second)
20	100.5
30	91.0
40	84.6
50	80.5
60	77.0
70	74.4
80	77.0
90	69.8
100	68.0
110+	67.0

Class C f/b* Class B

λ_s	T (second)
20	71.0
30	63.5
40	58.5
50	55.0
60	52.2
70	50.0
80	48.0
90	46.5
100	45.5
110+	44.0

TABLE 4-II (cont)

Class C f/b* λ_s	Class C T (second)
20	54.5
30	53.8
40	44.8
50	42.0
60	39.5
70	37.8
80	36.5
90	35.5
100	35.0
110+	35.0

Class C f/b* λ_s	Classes D and E T (second)
20	54.4
30	45.5
40	39.6
50	36.0
60	33.3
70	31.2
80	31.0
90	31.0
100	31.0
110+	31.0

Classes D and E f/b* λ_s	Class A T (second)
20	94.5
30	90.3
40	87.8
50	85.5
60	83.6
70	82.2
80	81.2
90	80.0
100	79.0
110+	78.3

Classes D and E f/b* λ_s	Class B T (second)
20	59.0
30	55.5
40	52.5
50	50.5
60	49.0
70	48.0
80	46.6
90	46.0
100	46.0
110+	46.0

Classes D and E f/b* λ_s	Class C T (second)
20	68.0
30	53.8
40	45.5
50	40.0
60	36.0
70	34.0
80	34.0
90	34.0
100	34.0
110+	34.0

Classes D and E f/b* λ_s	Classes D and E T (second)
20	53.5
30	44.0
40	38.2
50	34.0
60	31.0
70	28.8
80	27.0
90	25.4
100	24.0
110+	23.0

TABLE 4-III

TIME FROM "CLEAR TO TAKEOFF" TO "START ROLL" FOR DEPARTURES

Aircraft Class	Time (second)
A	18
B	9
C	8
D	4
E	4

TABLE 4-IV

T, AVERAGE MINIMUM SPACING BETWEEN SUCCESSIVE DEPARTURES ON SAME RUNWAY AND SAME DEPARTURE ROUTE (IFR)

Class A f/b* Class A

λ_s	T (second)
10	94.2
20	90.0
30	87.5
40	86.0
50	86.0
60+	86.0

Class A f/b* Class B

λ_s	T (second)
10	85.2
20	81.2
30	79.0
40	77.0
50	77.0
60+	77.0

Class A f/b* Class C

λ_s	T (second)
10	100.8
20	83.2
30	74.8
40	69.4
50	67.0
60+	67.0

Class A f/b* Classes D & E

λ_s	T (second)
10	96.8
20	85.5
30	78.5
40	73.5
50	70.4
60+	67.5

Class B f/b* Class A

λ_s	T (second)
10	114.5
20	107.0
30	102.5
40	100.0
50	100.0
60+	100.0

Class B f/b* Class B

λ_s	T (second)
10	110.0
20	89.0
30	78.5
40	72.0
50	69.0
60+	69.0

T = average minimum spacing between successive departures.

λ_s = movement rate.

* f/b = followed by.

TABLE 4-IV (cont)

Class B f/b* Class C

λ_s	T (second)
10	79.0
20	69.4
30	65.2
40	62.0
50	59.8
60+	59.0

Class B f/b* Classes D & E

λ_s	T (second)
10	108.3
20	77.5
30	63.8
40	55.8
50	49.5
60+	45.5

Class C f/b* Class A

λ_s	T (second)
10	145.5
20	126.4
30	117.3
40	111.0
50	111.0
60+	111.0

Class C f/b* Class B

λ_s	T (second)
10	129.0
20	113.6
30	106.0
40	101.0
50	101.0
60+	101.0

Class C f/b* Class C

λ_s	T (second)
10	97.0
20	84.5
30	78.0
40	74.0
50	73.0
60+	73.0

Class C f/b* Classes D & E

λ_s	T (second)
10	103.8
20	84.2
30	75.5
40	69.5
50	67.0
60+	67.0

Classes D and E f/b* Class A

λ_s	T (second)
10	157.0
20	150.2
30	146.0
40	143.0
50	143.0
60+	143.0

Classes D and E f/b* Class B

λ_s	T (second)
10	143.0
20'	136.0
30	131.5
40	129.0
50	129.0
60	129.0

TABLE 4-IV (cont)

Classes D and E f/b* λ_s	Class C T (second)	Classes D and E f/b* λ_s	Classes D & E T (second)
10	133.8	10	114.5
20	100.0	20	91.2
30	100.0	30	81.8
40	100.0	40	76.0
50	100.0	50	75.0
60+	100.0	60+	75.0

TABLE 4-V

T, AVERAGE MINIMUM SPACING BETWEEN SUCCESSIVE DEPARTURES ON SAME RUNWAY BUT ON DIFFERENT DEPARTURE ROUTES (IFR)

Class A f/b* Class A

λ_s	T (second)
10	76.0
20	72.0
30	69.8
40	68.0
50	67.0
60+	65.8

Class A f/b* Class B

λ_s	T (second)
10	83.2
20	79.0
30	76.8
40	75.0
50	73.8
60+	72.8

Class A f/b* Class C

λ_s	T (second)
10	100.8
20	83.2
30	74.8
40	69.4
50	65.5
60+	62.5

Class A f/b* Classes D & E

λ_s	T (second)
10	96.8
20	85.5
30	78.5
40	73.5
50	70.4
60+	67.5

Class B f/b* Class A

λ_s	T (second)
10	100.5
20	93.0
30	88.5
40	86.0
50	86.0
60+	86.0

Class B f/b* Class B

λ_s	T (second)
10	102.6
20	81.5
30	71.0
40	64.5
50	60.0
60+	56.2

T = average minimum spacing between successive departures.

λ_s = movement rate.

* f/b = followed by.

TABLE 4-V (cont)

Class B f/b* Class C

λ_s	T (second)
10	68.5
20	59.0
30	54.6
40	51.5
50	49.2
60+	47.5

Class B f/b* Classes D & E

λ_s	T (second)
10	108.3
20	77.5
30	63.8
40	55.8
50	49.5
60+	45.5

Class C f/b* Class A

λ_s	T (second)
10	125.5
20	106.3
30	97.3
40	91.0
50	91.0
60+	91.0

Class C f/b* Class B

λ_s	T (second)
10	97.5
20	82.0
30	74.4
40	69.6
50	67.0
60+	67.0

Class C f/b* Class C

λ_s	T (second)
10	80.5
20	68.0
30	61.6
40	57.4
50	55.0
60+	52.6

Class C f/b* Classes D & E

λ_s	T (second)
10	90.0
20	70.8
30	62.0
40	56.0
50	52.8
60+	50.0

Classes D and E f/b* Class A

λ_s	T (second)
10	131.2
20	124.3
30	120.0
40	117.2
50	116.0
60+	116.0

Classes D and E f/b* Class B

λ_s	T (second)
10	104.2
20	96.8
30	92.6
40	90.0
50	90.0
60+	90.0

TABLE 4-V (cont)

Classes D and E f/b* λ_s	Class C T (second)	Class D and E f/b* λ_s	Classes D & E T (second)
10	114.2	10	94.0
20	80.8	20	71.0
30	68.0	30	61.2
40	68.0	40	55.5
50	68.0	50	52.0
60+	68.0	60+	52.0

TABLE 4-VI

ABSOLUTE MINIMUM VALUES OF F FOR SAME RUNWAY (VFR)

Aircraft Class (Departure)		Aircraft Class (Arrival)	F_{min} (sec)
A	followed by	A	51
A		B	60
A		C	64
A		D	67
A		E	75
B		A	38
B		B	32
B		C	36
B		D	39
B		E	45
C		A	39
C		B	29
C		C	29
C		D	32
C		E	40
D		A	38
D		B	30
D		C	24
D		D	20
D		E	22
E		A	38
E		B	30
E		C	24
E		D	20
E		E	21

F = departure release in front of an incoming arrival.

TABLE 4-VII

AVERAGE MINIMUM VALUES OF F FOR SAME RUNWAY (IFR)

Aircraft Class (Departure)		Aircraft Class (Arrival)	F (sec)
A	followed by	A	56
A		B	74
A		C	83
A		D	66
A		E	86
B		A	56
B		B	43
B		C	50
B		D	66
B		E	86
C		A	56
C		B	43
C		C	50
C		D	66
C		E	86
D		A	56
D		B	43
D		C	50
D		D	66
D		E	86
E		A	56
E		B	43
E		C	50
E		D	66
E		E	86

F = departure release in front of an incoming arrival.

TABLE 4-VIII

AVERAGE TIME FROM OVER-THRESHOLD TO RUNWAY TOUCHDOWN FOR ARRIVALS IN VFR (EQUALS VALUE OF R FOR OPEN-V RUNWAYS)

	Runway Length (feet)					
Aircraft Class	to 5300	5301 to 6199	6200 to 7000	7001 to 9500	9501 to 12,999	≥13,000
A		6.0	7.0	7.0	7.5	8.0
B	5.0	6.0	7.0	8.0	8.0	8.0
C	5.0	7.0	8.5	10.5	10.5	10.5
D	5.0	7.0	8.5	10.5	10.5	10.5
E	5.0	8.0	11.0	14.0	14.0	14.0

All times in seconds.

R = runway occupancy for arrivals.

TABLE 4-IX

AVERAGE TIME FROM OVER-THRESHOLD TO RUNWAY TOUCHDOWN FOR ARRIVALS IN IFR (EQUALS VALUE OF R FOR OPEN V AND CLOSE PARALLEL RUNWAYS)

	Runway Length (feet)					
Aircraft Class	to 5300	5301 to 6199	6200 to 7000	7001 to 9500	9501 to 12,999	≥13,000
A		8.0	10.0	10.0	10.0	11.0
B	10.0	12.0	14.0	17.0	17.0	17.0
C	14.0	16.0	18.0	20.0	20.0	20.0
D	15.0	16.0	18.0	20.0	20.0	20.0
E	21.0	24.0	26.0	29.0	29.0	29.0

All time in seconds.

R = runway occupancy for arrivals.

TABLE 4-X

A, AVERAGE MINIMUM SPACING BETWEEN SUCCESSIVE ARRIVALS (VFR)

Class A f/b* Class A

λ_L	A (second)
10	94.0
20	87.0
30	83.0
40	80.2
50	78.5
60+	77.0

Class A f/b* Class B

λ_L	A (second)
10	91.5
20	86.0
30	83.0
40	80.8
50	79.5
60+	78.0

Class A f/b* Class C

λ_L	A (second)
10	111.5
20	90.5
30	80.4
40	73.5
50	69.0
60+	66.0

Class A f/b* Classes D & E

λ_L	A (second)
10	94.4
20	80.4
30	73.0
40	68.5
50	64.5
60+	62.0

Class B f/b* Class A

λ_L	A (second)
10	102.0
20	89.0
30	82.0
40	77.5
50	74.3
60+	72.0

Class B f/b* Class B

λ_L	A (second)
10	113.0
20	79.0
30	64.3
40	54.5
50	50.0
60+	50.0

A = average minimum spacing between successive arrivals.

λ_L = arrival rate.

* f/b = followed by.

TABLE 4-X (cont)

Class B f/b* Class C

λ_L	A (second)
10	113.5
20	80.5
30	67.5
40	59.5
50	54.5
60+	50.0

Class B f/b* Classes D & E

λ_L	A (second)
10	110.8
20	70.8
30	53.5
40	44.5
50	39.0
60	39.0

Class C f/b* Class A

λ_L	A (second)
10	91.8
20	76.8
30	69.0
40	64.2
50	60.4
60	58.0

Class C f/b* Class B

λ_L	A (second)
10	75.5
20	63.0
30	57.0
40	53.0
50	50.4
60	48.0

Class C f/b* Class C

λ_L	A (second)
10	87.5
20	65.5
30	55.5
40	49.5
50	45.3
60	42.0

Class C f/b* Classes D & E

λ_L	A (second)
10	89.0
20	66.5
30	56.0
40	49.5
50	46.0
60	46.0

Classes D and E f/b* Class A

λ_L	A (second)
10	88.6
20	64.8
30	54.2
40	50.0
50	50.0
60	50.0

Classes D and E f/b* Class B

λ_L	A (second)
10	78.0
20	61.5
30	53.8
40	48.8
50	47.0
60	47.0

TABLE 4-X

A, AVERAGE MINIMUM SPACING BETWEEN SUCCESSIVE ARRIVALS (VFR)

Class A f/b* Class A

λ_L	A (second)
10	94.0
20	87.0
30	83.0
40	80.2
50	78.5
60+	77.0

Class A f/b* Class B

λ_L	A (second)
10	91.5
20	86.0
30	83.0
40	80.8
50	79.5
60+	78.0

Class A f/b* Class C

λ_L	A (second)
10	111.5
20	90.5
30	80.4
40	73.5
50	69.0
60+	66.0

Class A f/b* Classes D & E

λ_L	A (second)
10	94.4
20	80.4
30	73.0
40	68.5
50	64.5
60+	62.0

Class B f/b* Class A

λ_L	A (second)
10	102.0
20	89.0
30	82.0
40	77.5
50	74.3
60+	72.0

Class B f/b* Class B

λ_L	A (second)
10	113.0
20	79.0
30	64.3
40	54.5
50	50.0
60+	50.0

A = average minimum spacing between successive arrivals.

λ_L = arrival rate.

* f/b = followed by.

TABLE 4-X (cont)

Class B f/b* Class C

λ_L	A (second)
10	113.5
20	80.5
30	67.5
40	59.5
50	54.5
60+	50.0

Class B f/b* Classes D & E

λ_L	A (second)
10	110.8
20	70.8
30	53.5
40	44.5
50	39.0
60	39.0

Class C f/b* Class A

λ_L	A (second)
10	91.8
20	76.8
30	69.0
40	64.2
50	60.4
60	58.0

Class C f/b* Class B

λ_L	A (second)
10	75.5
20	63.0
30	57.0
40	53.0
50	50.4
60	48.0

Class C f/b* Class C

λ_L	A (second)
10	87.5
20	65.5
30	55.5
40	49.5
50	45.3
60	42.0

Class C f/b* Classes D & E

λ_L	A (second)
10	89.0
20	66.5
30	56.0
40	49.5
50	46.0
60	46.0

Classes D and E f/b* Class A

λ_L	A (second)
10	88.6
20	64.8
30	54.2
40	50.0
50	50.0
60	50.0

Classes D and E f/b* Class B

λ_L	A (second)
10	78.0
20	61.5
30	53.8
40	48.8
50	47.0
60	47.0

TABLE 4-X (cont)

Classes D and E f/b* Class C λ_L	A (second)	Classes D and E f/b* Classes D & E λ_L	A (second)
10	72.8	10	82.5
20	54.2	20	50.5
30	45.5	30	38.2
40	40.0	40	32.0
50	39.0	50	26.6
60	39.0	60	24.0

TABLE 4-XI

A, AVERAGE MINIMUM SPACING BETWEEN SUCCESSIVE ARRIVALS (IFR)

Class A f/b* Class A

λ_s	A (second)
10	179.0
20	172.0
30	168.0
40	165.0
50	164.0
60+	162.0

Class A f/b* Class B

λ_s	A (second)
10	190.0
20	184.0
30	181.0
40	179.0
50	177.0
60+	176.0

Class A f/b* Class C

λ_s	A (second)
10	220.0
20	200.0
30	189.0
40	182.0
50	178.0
60+	174.0

Class A f/b* Classes D & E

λ_s	A (second)
10	226.0
20	212.0
30	204.0
40	199.0
50	196.0
60	193.0

Class B f/b* Class A

λ_s	A (second)
10	136.0
20	123.0
30	116.0
40	111.0
50	108.0
60	106.0

Class B f/b* Class B

λ_s	A (second)
10	176.0
20	140.0
30	125.0
40	116.0
50	111.0
60	111.0

A = average minimum spacing between successive arrivals.

λ_s = movement rate.

* f/b = followed by.

TABLE 4-XI (cont)

Class B f/b* Class C

λ_s	A (second)
10	161.0
20	133.0
30	120.0
40	111.0
50	107.0
60	103.0

Class B f/b* Classes D & E

λ_s	A (second)
10	233.0
20	193.0
30	176.0
40	166.0
50	161.0
60	161.0

Class C f/b* Class A

λ_s	A (second)
10	144.0
20	129.0
30	122.0
40	117.0
50	113.0
60	110.0

Class C f/b* Class B

λ_s	A (second)
10	121.0
20	108.0
30	102.0
40	98.0
50	96.0
60	93.0

Class C f/b* Class C

λ_s	A (second)
10	160.0
20	138.0
30	129.0
40	122.0
50	118.0
60	115.0

Class C f/b* Classes D & E

λ_s	A (second)
10	184.0
20	161.0
30	151.0
40	145.0
50	141.0
60	141.0

Classes D and E f/b* Class A

λ_s	A (second)
10	136.0
20	112.0
30	101.0
40	97.0
50	97.0
60	97.0

Classes D and E f/b* Class B

λ_s	A (second)
10	139.0
20	122.0
30	114.0
40	109.0
50	108.0
60	108.0

TABLE 4-XI (cont)

Classes D and E f/b* λ_s	Class C A (second)	Classes D and E f/b* λ_s	Classes D & E A (second)
10	149.0	10	179.0
20	130.0	20	148.0
30	121.0	30	136.0
40	116.0	40	128.0
50	115.0	50	124.0
60	115.0	60	120.0

TABLE 4-XII

MODEL TESTING
ACTUAL VS COMPUTED DELAYS

FINAL PHASE TESTING

Airport	Date	Hours	IFR or VFR	λ_L	λ_T	Actual Delay (min)	Computed Delay (min)	Remarks
Wichita	9-22-59	6.9	VFR	13	12	0.2	0.5	Single runway departure delay
Miami	12-4-59	3.1	VFR	25	25	2.8	3.1	Predominantly single runway departure delay
Atlanta	12-1-59	1.8	VFR	26	39	1.4	1.8	3 intersecting runways departure delay
Washington	3-29-61	2.2	VFR	39	39	2.3	3.3	3 intersecting runways departure delay
Washington	3-31-61	4.7	IFR	26	27	3.4	3.4	3 intersecting runways departure delay
Washington	3-31-61	5.3	IFR	27	--	1.9	1.9	Arrivals only - FIM test on arrival delay
Idlewild	1-22-62	4.7 (a.m.)	IFR	13	--	1.2	1.1	Arrivals only - FIM test on arrival delay
Idlewild	1-22-62	3.7 (p.m.)	IFR	20	--	5.5	5.5	Arrivals only - FIM test on arrival delay

λ_L = Arrival rate.

λ_T = Movement rate.

TABLE 4-XIII

FIRST PHASE TESTING REPORTED IN REFERENCE 1

Airport	Date	Hours	IFR or VFR	λ_L	λ_T	Actual Delay (min)	Computed Delay (min)	Remarks
Wichita	9-22-59	6.9	VFR	14	12	0.2	0.6	Single runway, departure delay
Miami	12-4-59	3.1	VFR	25	25	2.8	4.6	Predominantly single runway departure delay
Idlewild	4-4-58	8.2	VFR	--	19	1.3	1.5	Departures only. FIM test on departure delay
Newark A	3-27-58	10.2	VFR	9	10	1.2	1.0	Single runway, departure delay
B	3-27-58	2.7	VFR	13	20	1.9	2.2	Single runway, departure delay
LaGuardia	9-4-59	5.1	VFR	31	35	4.5	4.4	Single runway, departure delay
LaGuardia	11-13-59	3.1	VFR	22	24	2.2	1.8	Single runway, departure delay

λ_L = Arrival rate.

λ_T = Movement rate.

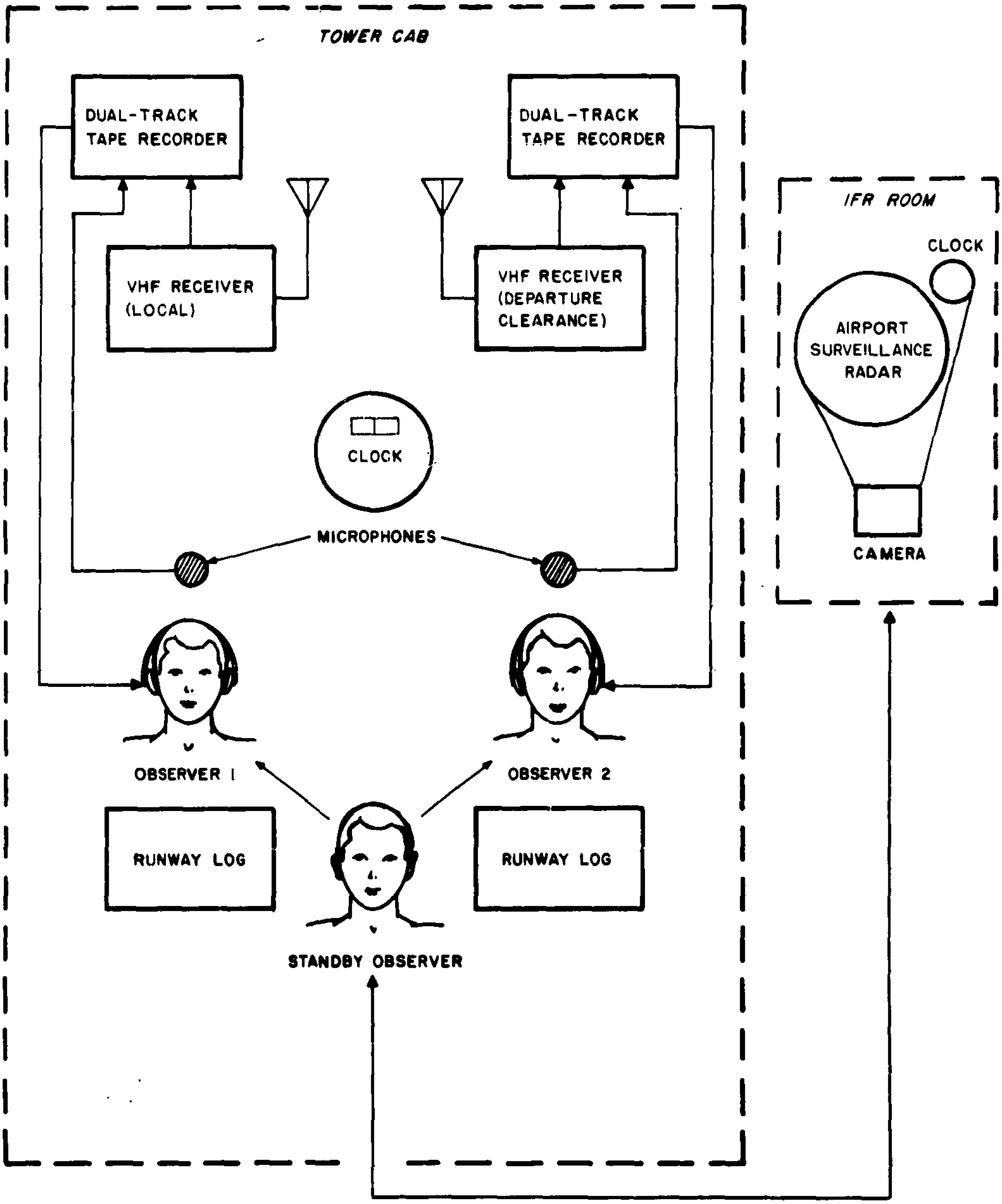


FIGURE 4-1. AIRPORT SURVEY RECORDING TECHNIQUE

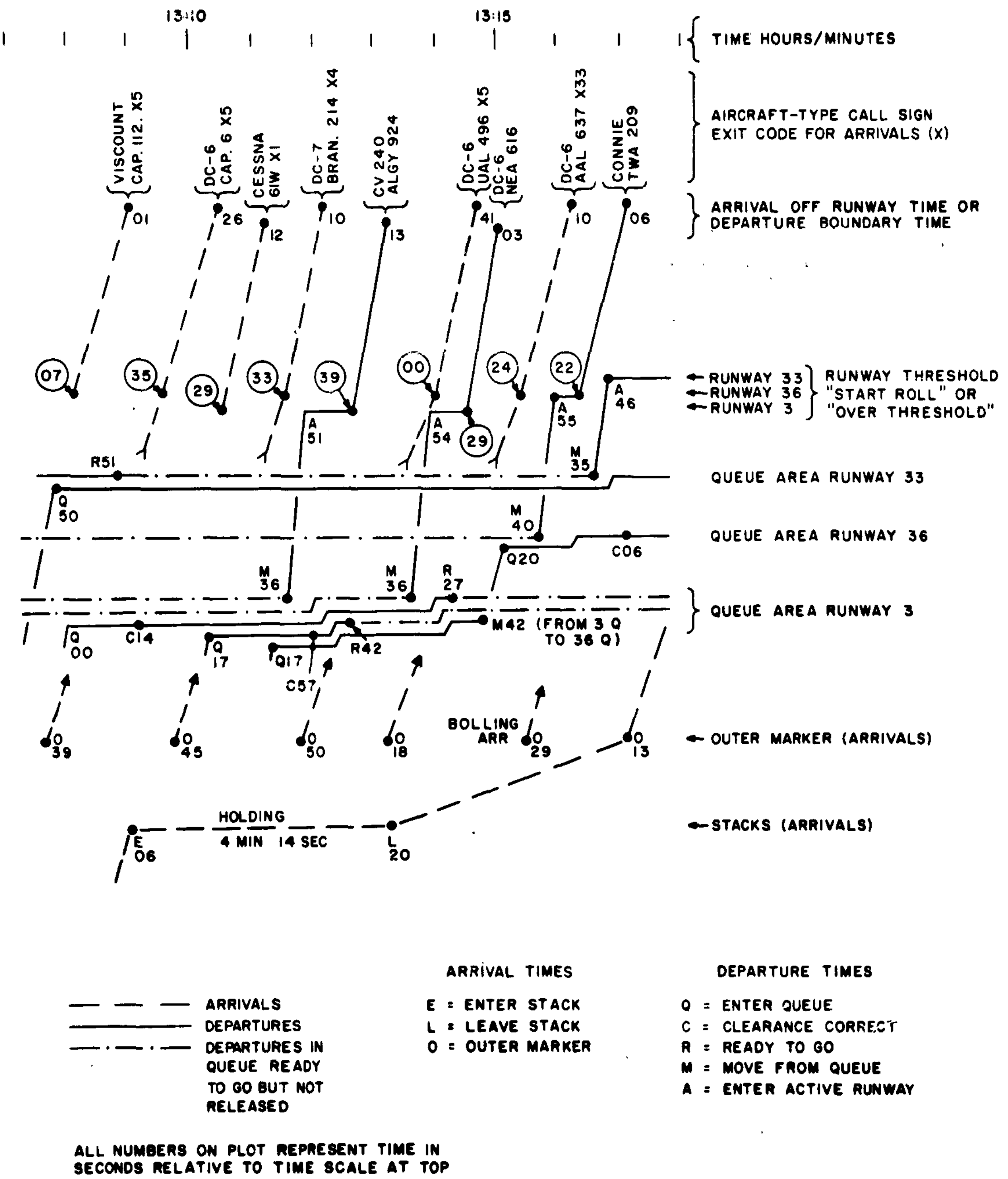


FIGURE 4-2. EXAMPLE OF AIRPORT DATA PLOT

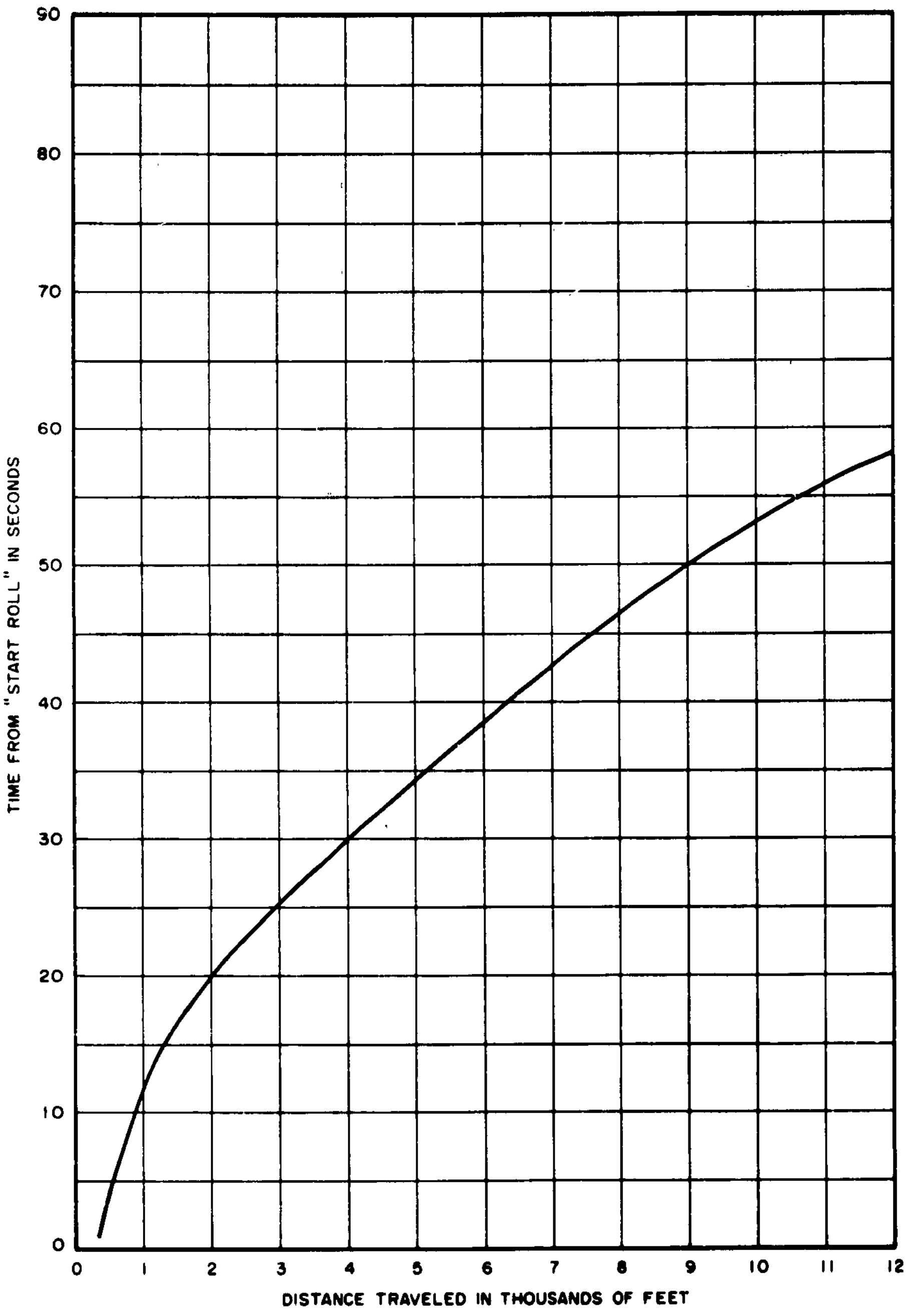


FIGURE 4-3. DISTANCE VS TIME FOR TAKEOFF, CLASS A

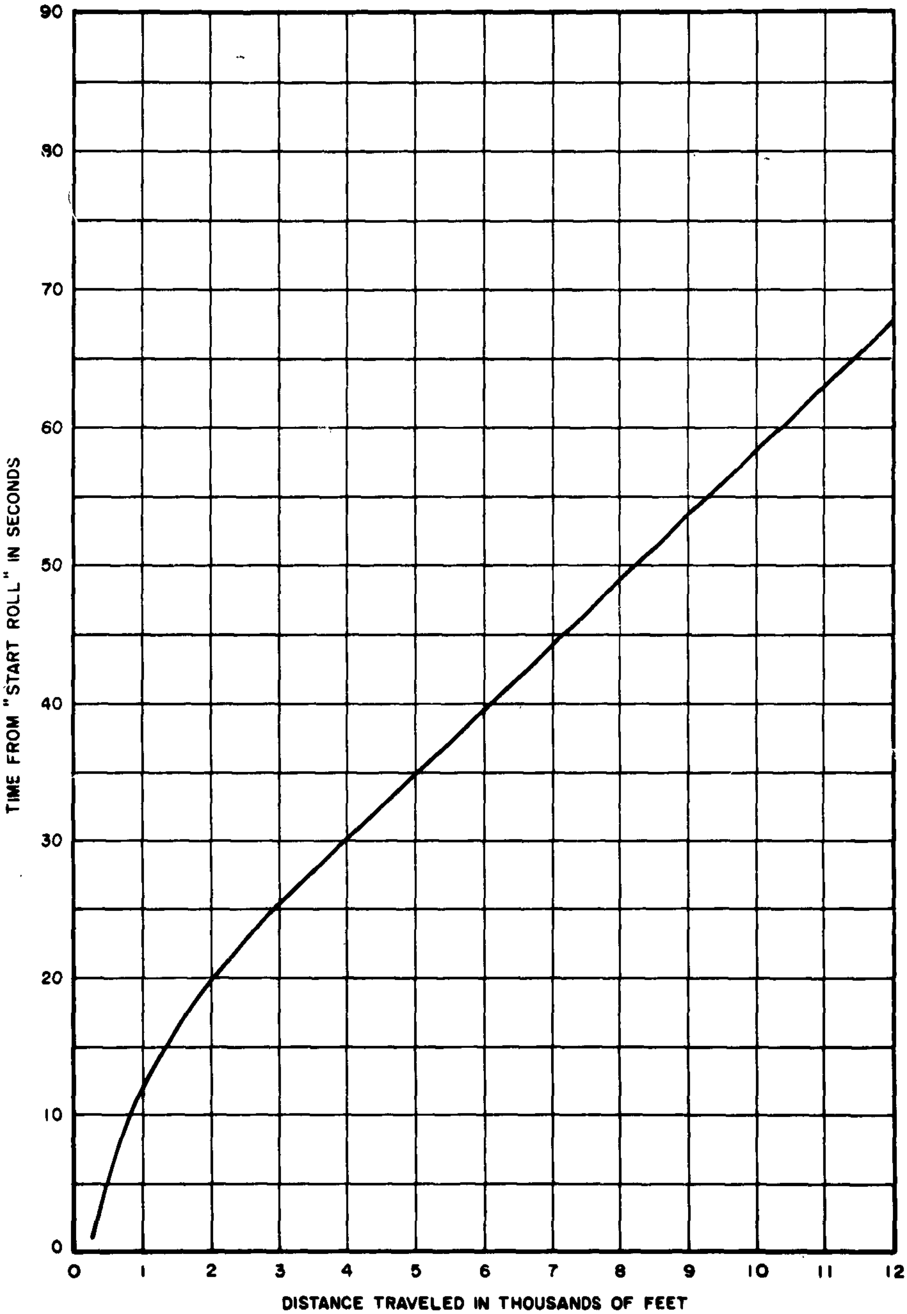


FIGURE 4-4. DISTANCE VS TIME FOR TAKEOFF, CLASS B

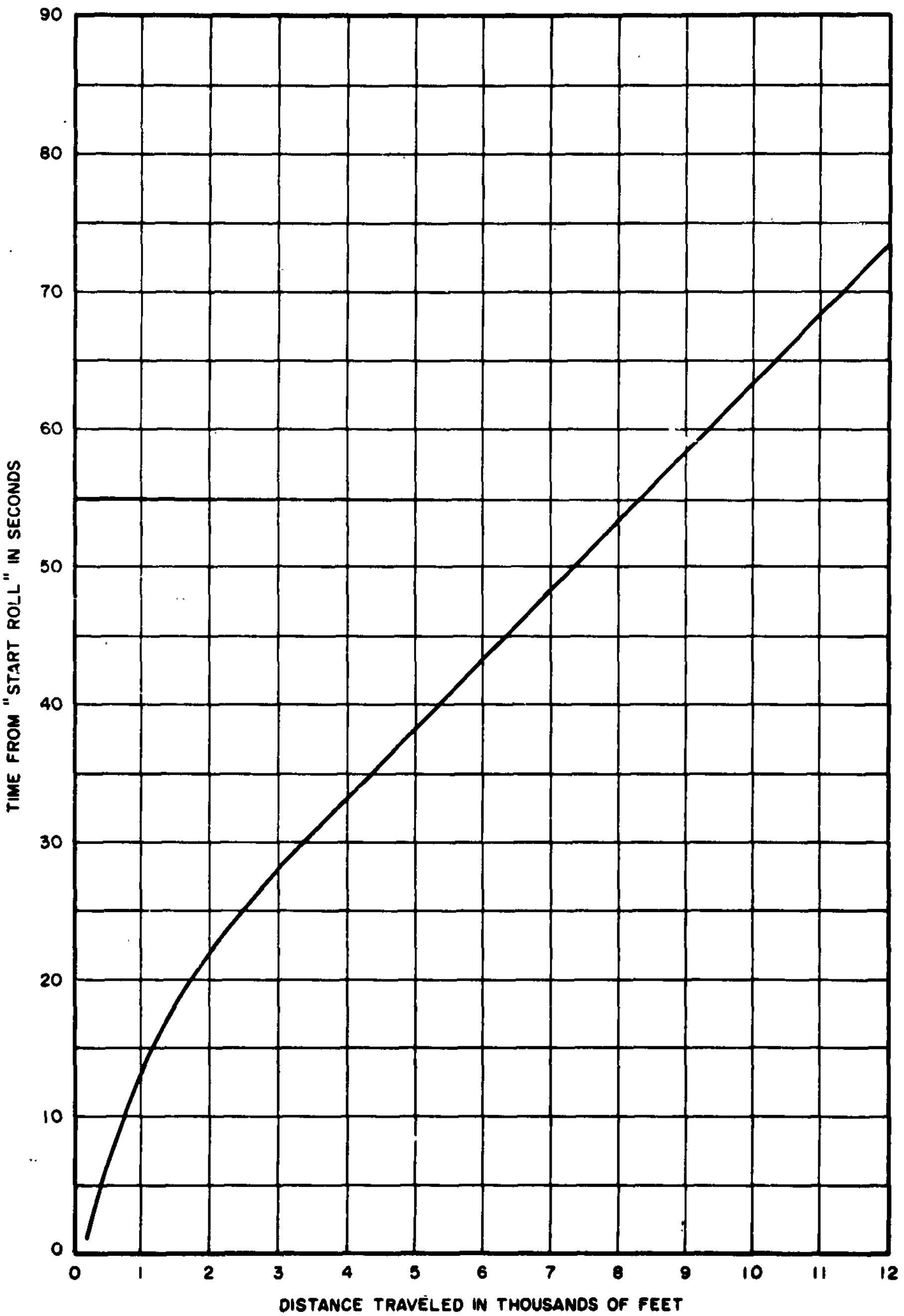


FIGURE 4-5. DISTANCE VS TIME FOR TAKEOFF, CLASS C

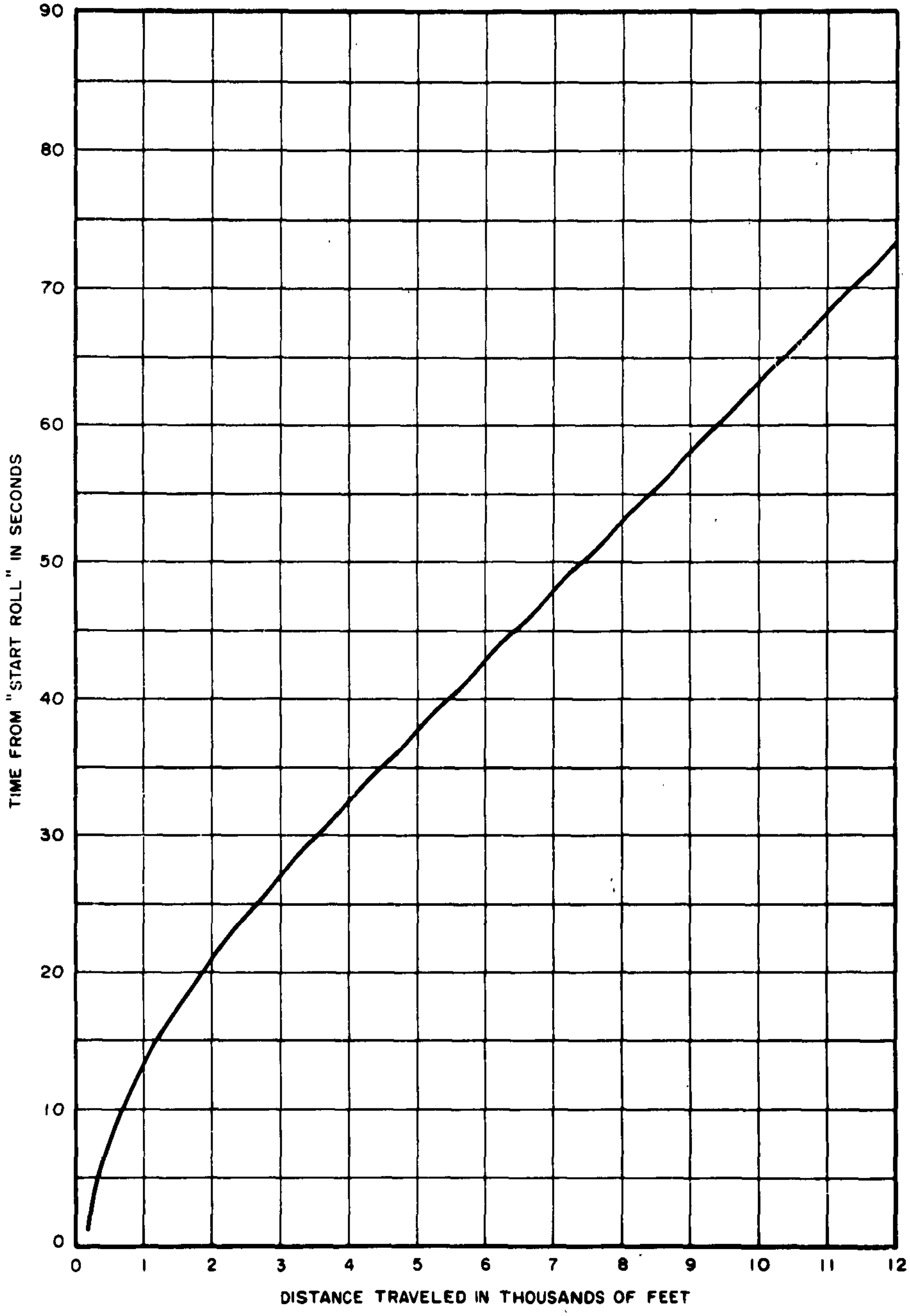


FIGURE 4-6. DISTANCE VS TIME FOR TAKEOFF, CLASS D

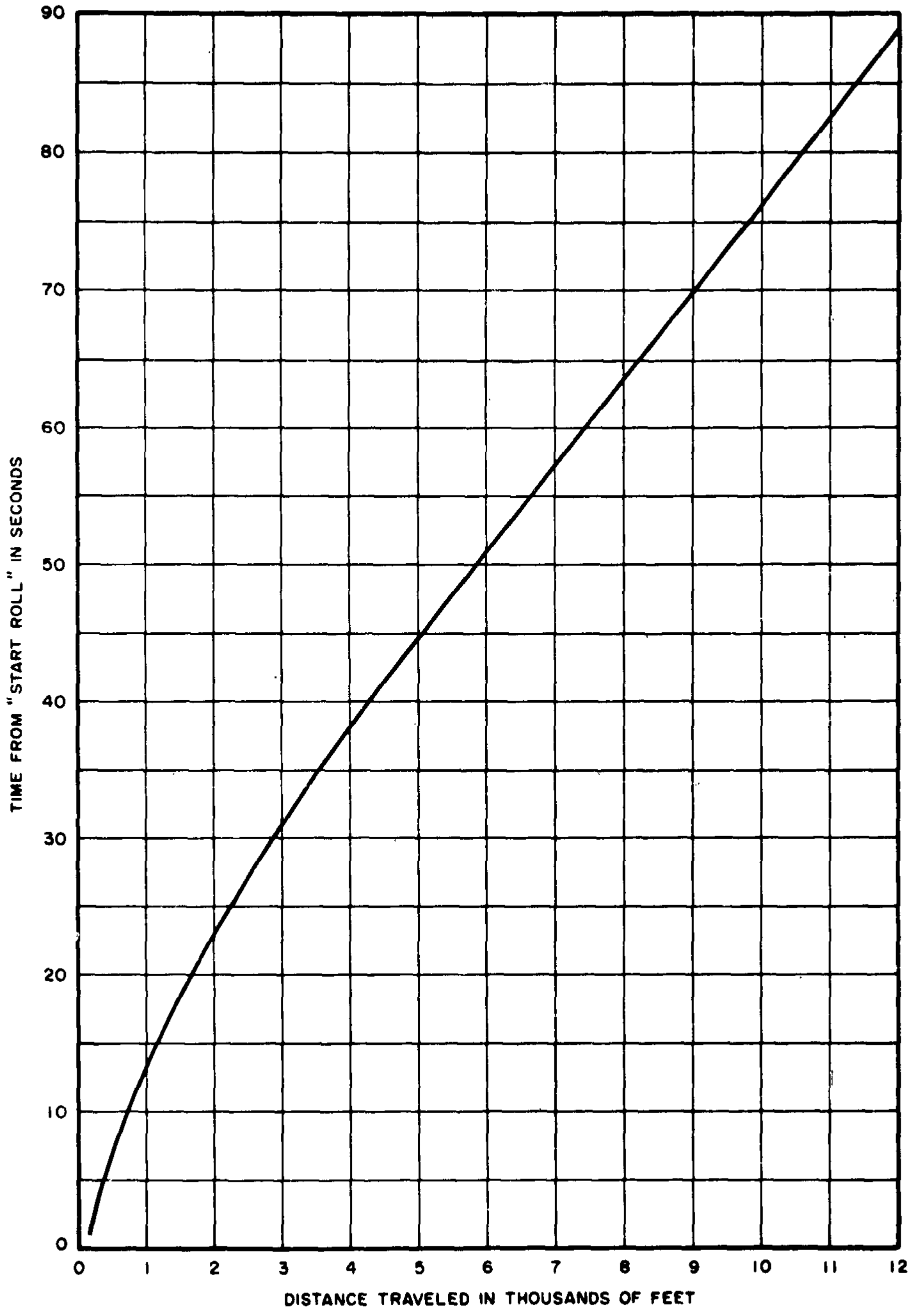


FIGURE 4-7. DISTANCE VS TIME FOR TAKEOFF, CLASS E

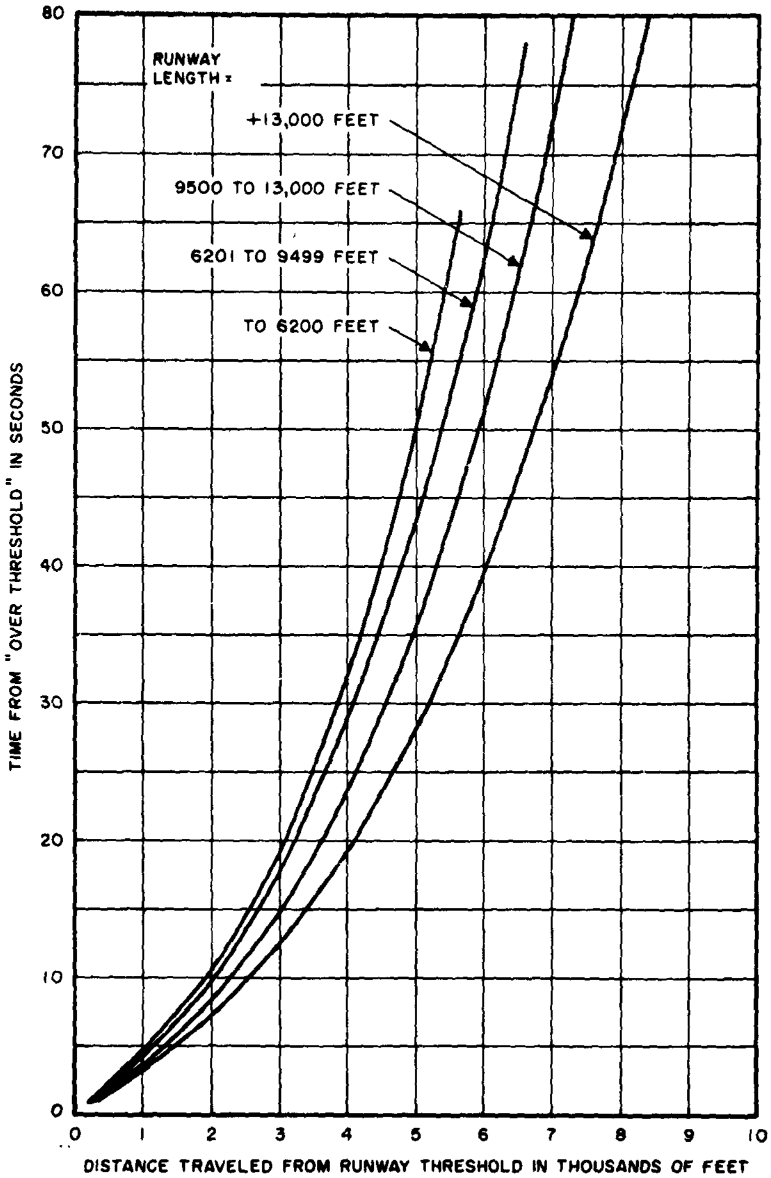


FIGURE 4-8. DISTANCE VS TIME FOR LANDING, CLASS A

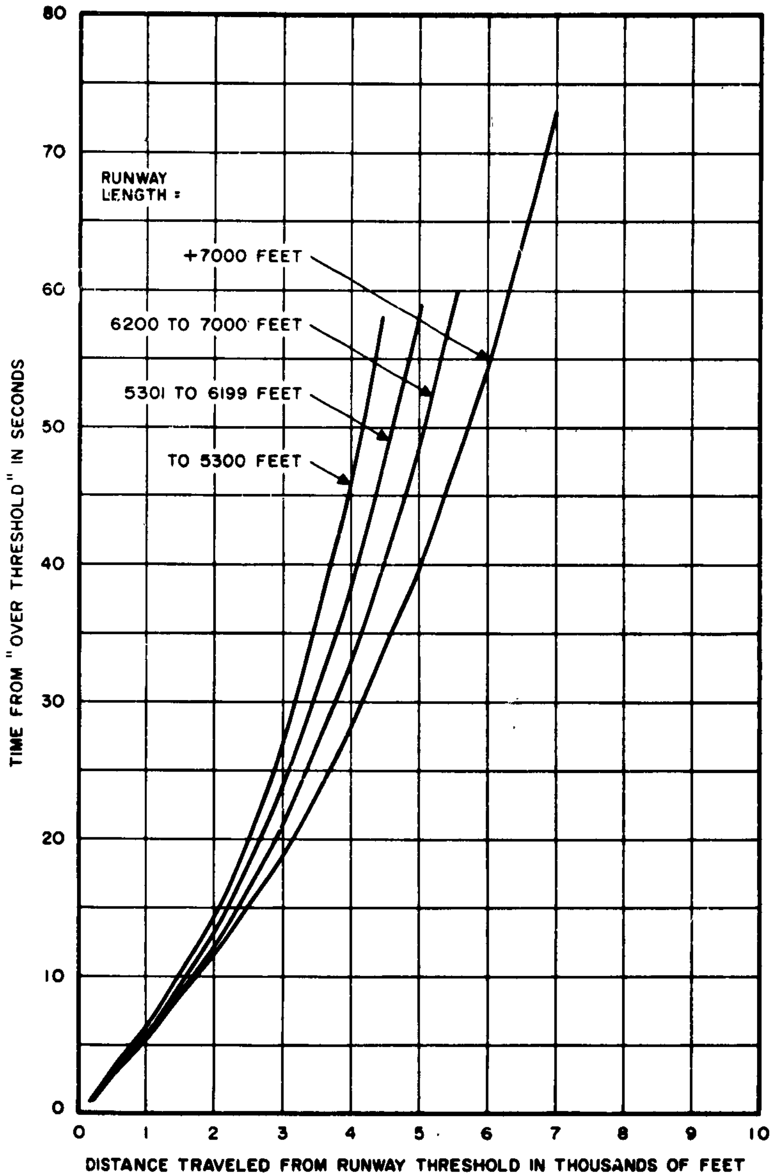


FIGURE 4-9. DISTANCE VS TIME FOR LANDING, CLASS B

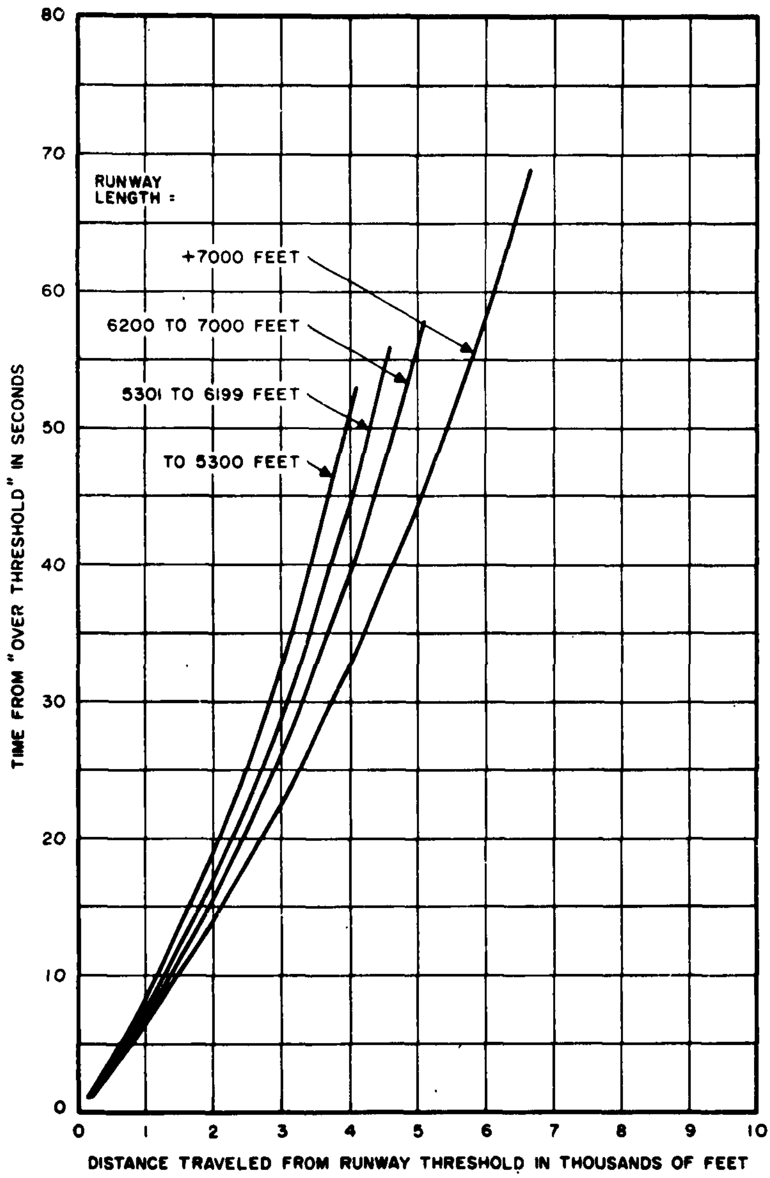


FIGURE 4-10. DISTANCE VS TIME FOR LANDING, CLASS C

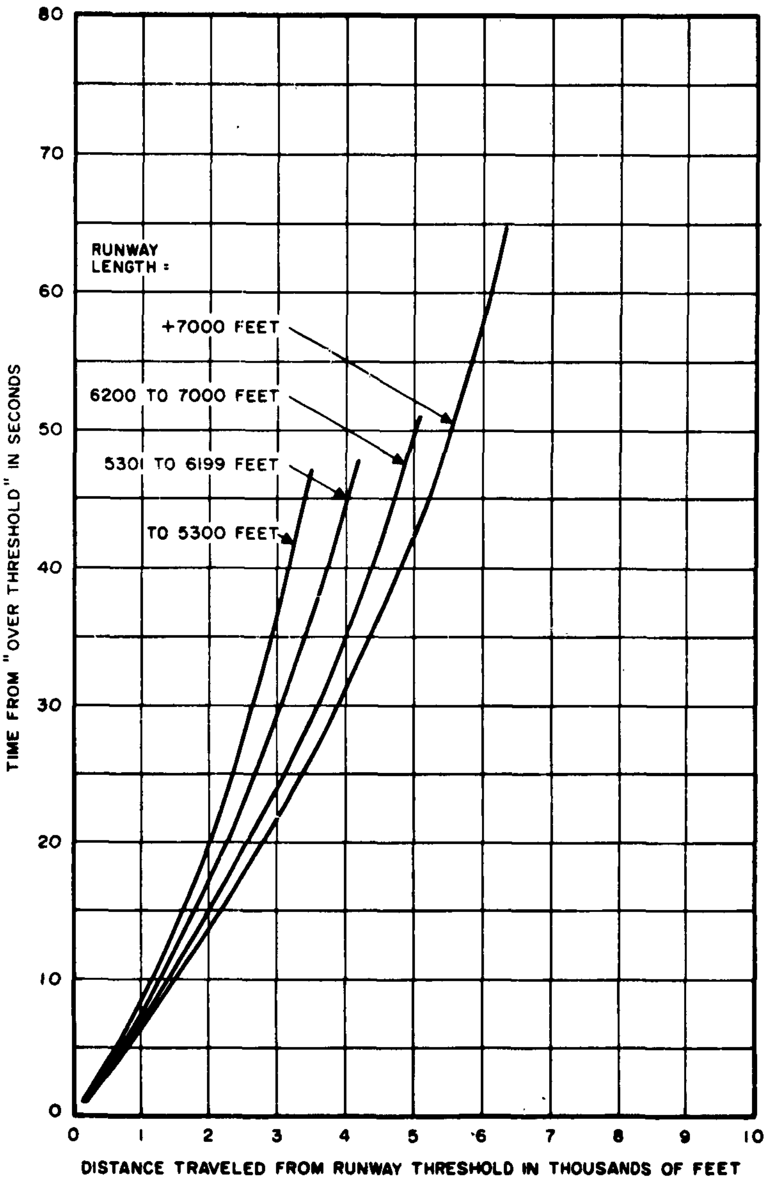


FIGURE 4-11. DISTANCE VS TIME FOR LANDING, CLASS D

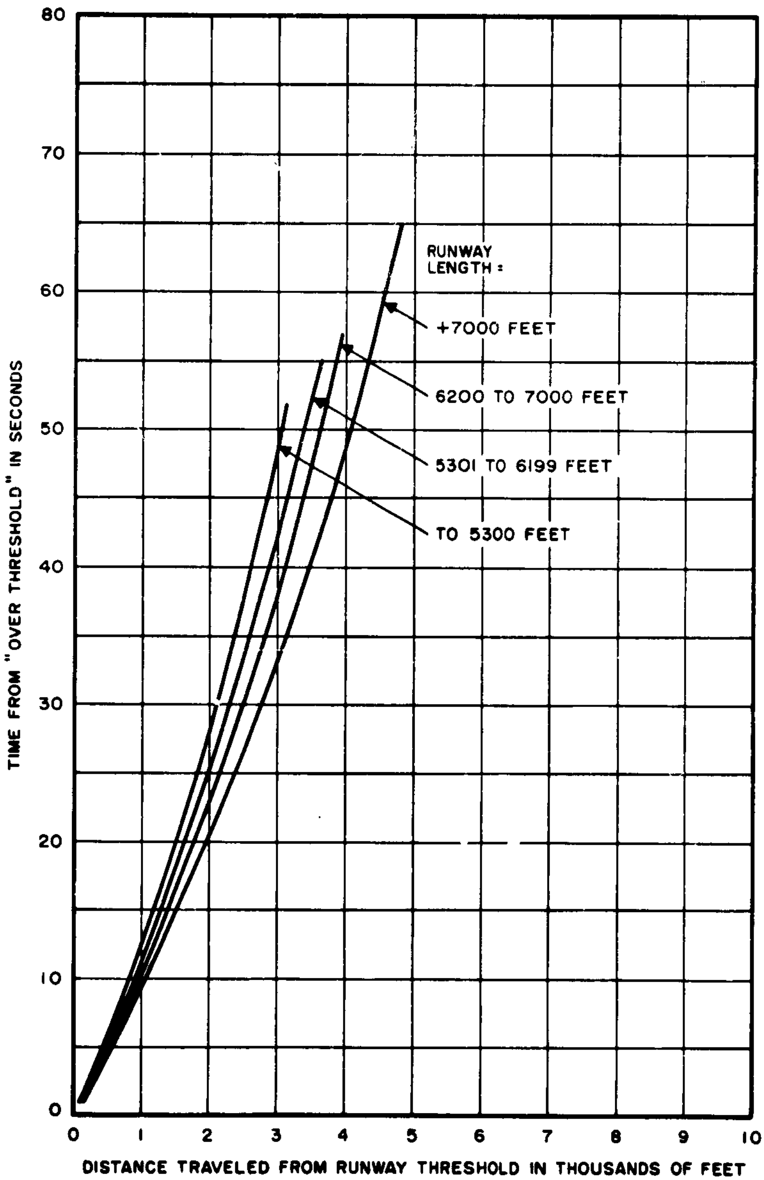


FIGURE 4-12. DISTANCE VS TIME FOR LANDING, CLASS E

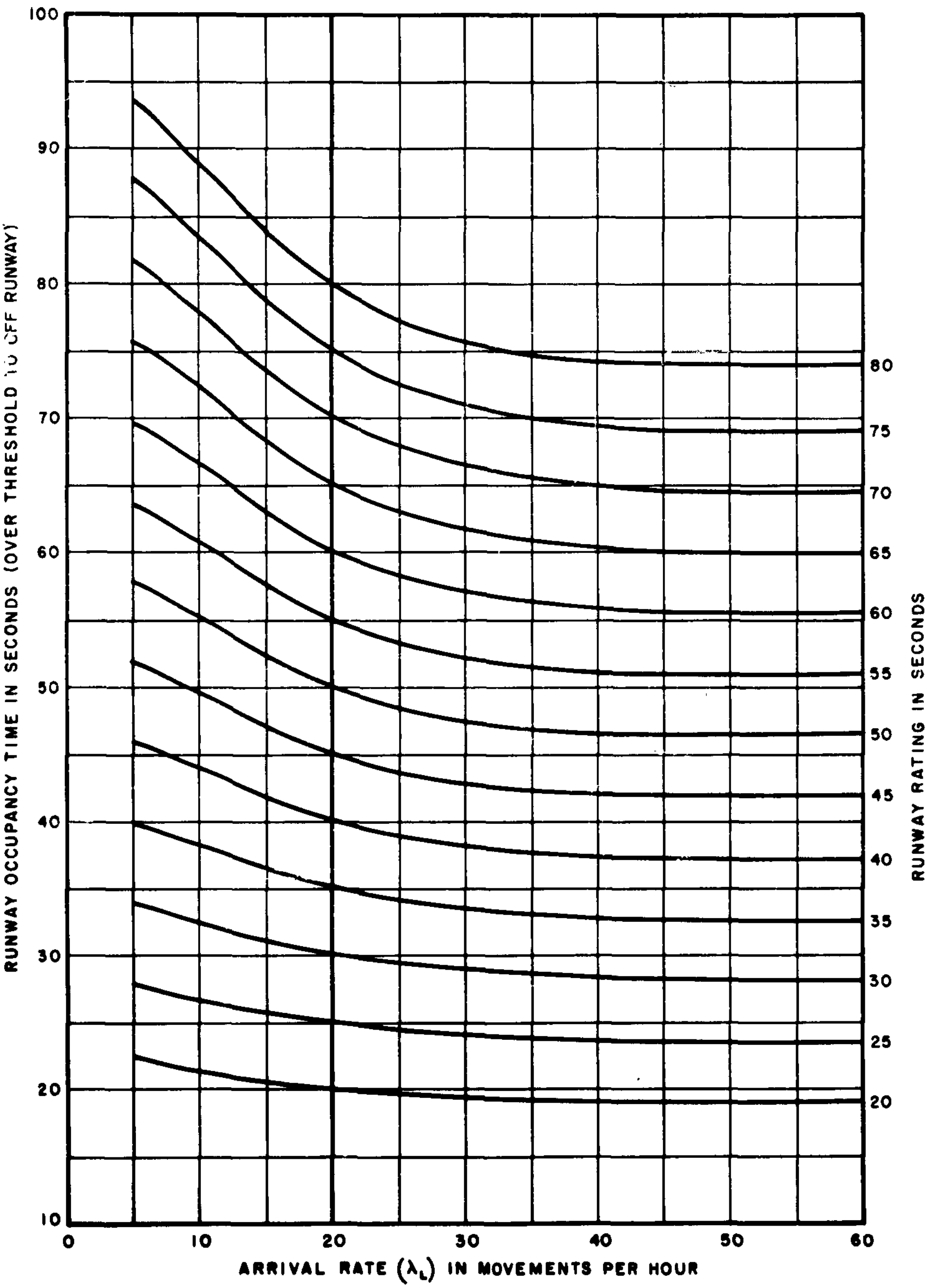


FIGURE 4-13. RUNWAY RATING CURVES

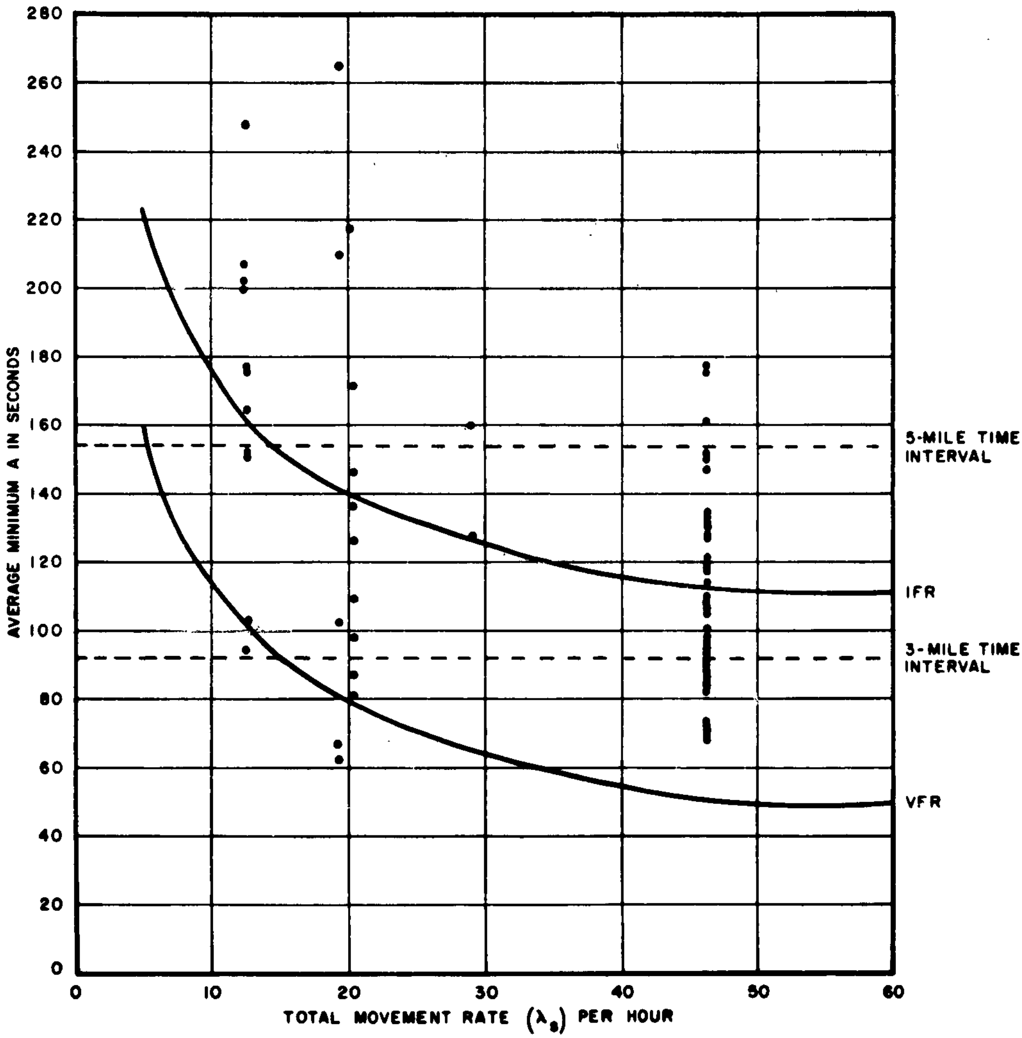


FIGURE 4-14. SAMPLE DATA FROM SURVEY, INTERVAL A (IFR)

V. DESCRIPTION OF MATHEMATICAL MODELS

A. GENERAL

This section deals exclusively with the mathematical description of the model and the inputs. The practical aspects were covered in Sections II, III, and IV.

Since the previous work, the Pre-emptive Poisson Arrivals Model (PAM) has been completely discarded because analysis of the runway/taxiway crossing problem has indicated that a special application of SAM is more practical (Appendix F). Therefore, only SAM and FIM will be covered here.

Because there have been a number of changes in the notation since the previous work, a new glossary of terms is included:

ARRIVALS

CL	Commitment to land
OT	Over threshold
OR	Off runway
R	Runway occupancy, OR - OT
L	Inter-arrival time for arrivals, OT(n) - OT(n-1)
A	Average minimal safe value of L
C	Commitment interval, OT - CL
B	Arrival service time, B = R + C
G	Arrival gaps, G = L - B

DEPARTURES

RG	Ready to go
CT	Cleared to takeoff
W	Departure delay, W = CT - RG
D	Inter-departure time for departures, CT(n) - CT(n-1)
T	Average minimal safe value of D (constant)
F	Average minimal value of G (between arrivals) to permit CT (of departure) (constant)
K	Interval that starts when n-1 departure takes off
H	Interval that starts at the end of K
J	J = H + K
FR	CT(n-1) + J(n)

RATES

λ_L = Landing rate, $\frac{1}{\text{average L}}$

λ_T = Takeoff rate, $\frac{1}{\text{average D}}$

$\lambda_S = \lambda_L + \lambda_T$

FIM

Used for calculating delay when there are (1) arrivals only, or (2) departures only.

Average arrival delay is

$$\frac{\lambda_L(a_2)}{2\left[1 - \lambda_L(a_1)\right]}$$

Average departure delay is

$$\frac{\lambda_T\ (t_2)}{2\left[1 - \lambda_T\ (t_1)\right]}$$

where

a_1 or t_1 = average value (first moment)
a_2 or t_2 = second moment.

SAM

The model and the backup work performed to make SAM a practical tool can be better understood if the following facts are noted:

1. It permits a variety of specializations, each of which can make it apply to an individual element of the airport design, such as a single runway, a complex of interdependent runways, a runway/taxiway crossing--each under various operating specifications. There are two main subtypes of each specialization: (1) some component of the traffic has priority over the remainder--for example, arrivals over departures, or runway traffic over taxiway crossing traffic (SAM), (2) in addition, there is a special subtype for priority traffic when that traffic tends to be singularly clustered in its time pattern of flow (PAM).

2. The model treats delay as a probabilistic or chance phenomenon. Although individual delays are characterized only by the frequency of occurrence of given amounts of delay, average delay is still a basic measure of delay. The model includes a simple formula for average delay computed out of all the frequencies of all the various amount of delay incurred. Formulas for the various delay frequencies are more complicated.

3. The model is operational--that is, inputs are derived from measurements of real-life situations. It then computes delay by a detailed accounting of the accumulation of the large number of time elements as this accumulation grows and decays moment by moment in the course of actual operation of the runways during a period of time.

The basis of the model operation is that it seeks gaps (G) in the arrival sequence in which to release departures. The greater the number of arrivals the less the number of gaps and the greater will be the departure delay.

B. FORMULATION OF DELAY

From the mathematical standpoint, certain basic notions and quantities are common to the phenomenon of traffic delay at any of the several points where such delay may occur on the surface of the airport. To exhibit these elements of the formulation of delay, we present first a full discussion of delays to aircraft taking off at a single isolated runway used for both landings and takeoffs. Afterwards we shall show how the formulas developed for this case may be reinterpreted or modified to describe delays at other points on the surface of the airport. In this discussion some aspects of the inputs already covered in Sections II, III, and IV must necessarily be repeated for the sake of clarity and to illustrate the logic.

1. DELAYS TO TAKEOFFS AT SINGLE ISOLATED RUNWAY USED FOR LANDINGS AND TAKEOFFS

Consider any interval (t_1, t_2) of time during which the runway is in operation. There will be two sequences of aircraft to be served--namely, landings and takeoffs. Of the two, landings will normally be accorded priority over takeoffs for use of the runway because the landing aircraft is in

motion in the air at high speed and cannot be controlled physically to the extent possible for aircraft on the surface moving toward the takeoff point. Accordingly, we shall assume such priority to be uniformly preserved, and we term this degree of priority pre-emptive priority.

Suppose we number the aircraft in each of the two sequences according to its position in time in the sequence. Consider first the sequence of landings.

2. SEQUENCE OF LANDINGS

For the n^{th} aircraft in the sequence of landing, two times are of fundamental importance to the takeoff operation:

CL(n) = latest time at which the aircraft can be diverted from landing (waved off) as approaches touchdown if an obstacle is expected in its path on the runway ahead.

OR(n) = time at which the aircraft turns off the runway, releasing it for use by further traffic.

The time OR(n) is directly observable in any landing but the time CL(n) is not observable to an onlooker, though it may be a perfectly definite time for the pilot of the landing aircraft, and is, moreover required to be estimated by a controller within his mental process of deciding whether to release a takeoff in front of the oncoming landing. Its typical occurrence time may be estimated for various categories of aircraft with the assistance of a further time, which is directly observable for measurement, namely:

OT(n) = time at which the landing aircraft passes over the landing threshold of the runway.

The time CL(n) occurs before the time OT(n), and to a greater degree for aircraft that are unmaneuverable and have high landing speeds.

For the analysis of delay, certain intervals of time are of direct importance. For the n^{th} aircraft in a sequence of landings,

$OT(n)$ = time at which the aircraft passes over the landing threshold of the runway.

$OR(n)$ = time at which the aircraft turns off the runway, clearing the runway to following traffic.

For analysis of delay,

$$L(n) = OT(n) - OT(n-1)$$

This interval is termed the inter-arrival time for landings. The reciprocal of its average value is simply the arrival rate (in number of landings per unit time):

$$R(n) = OR(n) - OT(n)$$

This interval, whose importance is subsidiary to the interval to be discussed next, is simply the runway occupancy time of the n^{th} aircraft. The intervals $L(n)$ and $R(n)$ are directly measurable and very easily identified in actual operation. Less easily identified and measured, but of critical importance to delay computation, is the interval

$A(n)$ = the average minimal safe value of $L(n)$

As previously described, considerable portion of the measurement work supporting this report was devoted to determining the characteristic values of the intervals $A(n)$.

Since the intervals $A(n)$ are minimal time spacings between successive landings, the amount of delay to landings depends very much upon their lengths. We show this dependence

later. In the meantime, it should be remarked that the interval $A(n)$ is normally greater than the interval $R(n)$, and is long enough to permit waveoff of the n^{th} aircraft should the n-1 aircraft experience difficulty in landing and not appear to be able to turn off the runway in the normal amount of time.

An additional interval of importance to takeoffs as well as landings is the interval:

$$C(n) = OT(n) - CL(n)$$

We discuss this interval in paragraph 4 (Sequencing of Takeoffs Between Landings).

3. SEQUENCE OF TAKEOFFS

For the n^{th} aircraft in a sequence of takeoffs, the time at which the aircraft becomes ready to use the runway is denoted by

$RG(n)$ = "ready to go" time.

In principle, this time is slightly later than the time at which the pilot requests permission to use the runway, since the aircraft must in the meantime move into the runway. Thus, the times $RG(n)$ are takeoff demand times.

$CT(n)$ = time at which the aircraft is cleared for takeoff.

This interval must be similarly extrapolated if takeoff permission is granted before the aircraft enters the runway to obtain a correct accounting of the amount of delay the takeoff experiences. The time to move onto the runway is not

part of its delay. With these interpretations, we obtain the delay or wait of the n^{th} aircraft as simply:

$$W(n) = CT(n) - RG(n).$$

Among the other time intervals that contribute to W(n) there is in particular the minimal spacing interval between two takeoffs in succession, denoted as

T(n) = average minimal safe value of D(n)

where the inter-takeoff interval is

$$D(n) = CT(n) - CT(n-1)$$

The reciprocal of the average value of D(n) is simply the takeoff movement rate.

As was true of interval A(n) for landings, the intervals T(n) must be determined with care.

4. SEQUENCING OF TAKEOFFS BETWEEN LANDINGS

In addition to being required to be separated by an interval T(n) behind the n-1 aircraft at the time it receives takeoff clearance, the n^{th} takeoff must also be safely separated in front of an oncoming landing. Thus, at the time CT(n) occurs, there must exist a minimal separation time interval until the next oncoming landing reaches the beginning of its commitment interval.

The amount of time that must be specified for this separation significantly affects the delay to aircraft taking off. As a matter of actual observation, the separation interval available is too often just too short to allow a takeoff to be cleared safely. The basic safety requirement is that it be assured, once the takeoff has been cleared, that the

takeoff can be determined by the controller to be successful before the oncoming landing becomes committed to land--that is, before the beginning of the interval C for the oncoming landing.

In the course of the experimental measurement work supporting this study, a careful examination was made of the length of the interval C for various types of aircraft, and especially of the relationship between the length of C and A. An important result of this examination was the observation that the interval A is often larger than R + C. Thus, the sum of the intervals R and C is of fundamental importance in our analysis. Accordingly, we denote it specially as

$$B = R + C.$$

Our takeoff must be advanced so that its success can be established beforethe interval B commences. If the takeoff fails or aborts, it must be possible to wave off the oncoming landing. Consequently, CT(n) can only occur in the remaining portion of an interval L once the interval B is removed from L. Accordingly, this remainder or gap interval is also of basic importance, and we denote it by

$$G = L - B.$$

Moreover, our takeoff clearance CT(n) can only occur in a gap interval G of the landings pattern, and indeed only in such an interval G which is of sufficient length. That is G must be greater than F(n) where:

F(n) = minimal value of an interval G in which CT(n) can safely be given.

Thus, F(n) is a minimal spacing interval of a takeoff in front of a departure, measured from CT(n) to the beginning of the interval B for the next following landing.

As with the other spacing factors B and T, the intervals F must also be determined with care.

Exponential distribution of gap intervals. Perhaps the most remarkable fact noted in the measurement work was that under the wide variety of operating conditions actually observed and at all the types of airports observed, the gap intervals G(n) have a probability distribution that is nearly exponential for positive gaps. (This phenomenon, interestingly, enough, is also observable in the intervals between automobiles in a single traffic lane, and under a variety of roadways that includes both tunnels and high-speed freeways). This observation is supported by the theoretical fact that a random variable will tend to have an exponential distribution if its size is determinable by any one of a great number of causative factors, one of which in each case dominates all the others. For our purposes in this report, the principal test made of the supposition that these gaps intervals can be treated as having an exponential distribution is simply the fact that predictions of delay based upon that assumption are in accordance with the delays experimentally measured.

5. DETAILED ANALYSIS OF TAKEOFF DELAY

Interval K(n). When CT(n-1) occurs, it occurs in some interval G(j) of the landings sequence that is larger than F(n-1). We recall that G(j) will subsequently be followed by an interval B(j). When CT(n-1) occurs, there begins a subsidiary interval K(n) during which the n^{th} takeoff must

in any case be held. This interval is describable as follows:

$$K(n) = T(n) \quad \text{if } G(j) > T(n)$$
$$= G(j) + B(j) \text{ if } G(j) < T(n)$$

because the latter is then not long enough for two takeoffs. In single runway operation, it is almost always observable that if

$$G(j) \geq F(n-1)$$

then

$$G(j) + B(j) \geq T(n)$$

Accordingly, we shall assume this condition to be so (in cross runway traffic movements, the form of this assumption must be adapted carefully as we shall see later).

Interval H(n). The n^{th} takeoff can be released after the completion of the interval K(n) provided that it is released in some interval G [perhaps in the same interval G(j), should that interval be of sufficient length] of the landings sequence.

Consequently, upon termination of the interval K(n), there begins a further interval H(n) during which takeoff clearance may have to be withheld from the n^{th} takeoff, namely:

H(n) = time interval from the end of K(n) until in the landings sequence there first occurs an interval G which is greater than F(n).

Of course, H(n) will be 0 when G(j) is greater than T(n) plus F(n).

If we denote

$$J(n) = \text{interval } K(n) + H(n)$$
$$FR(n) = \text{time } CT(n-1) + J(n)$$

then, if RG(n) occurs before FR(n), we see that CT(n) will occur at FR(n).

Intervals V(n) and Z(n). However, if RG(n) occurs after FR(n), we must examine matters still further. Now FR(n) occurs in an interval G(i) of the landings process, where $i \geq j$, and $G(i) > F(n)$. Therefore, if RG(n) occurs immediately after FR(n), CT(n) will occur at RG(n). But if RG(n) then does not occur before FR(n) + G(i) - F(n), then an interval V(n) begins, where

$$V(n) = F(n) + B(i) + H(n)$$

during which the n^{th} takeoff will be held. This interval V(n) will be followed by an interval $G(k) > F(n)$, and then successively by another interval V(n), etc. Thus, FR(n) is followed by a sequence of intervals, G_1*, V_1, G_2*, V_2*, etc., where each G* is longer than F(n). If RG(n) occurs in one of the intervals G_1*, then CT(n) will occur at RG(n). But if RG(n) occurs in one of the intervals V_1, then CT(n) will occur at the end of that interval V_1.

Accordingly, let

Z(n) = interval, if RG(n) > FR(n), from RG(n) to the first interval G* in the landings process occurring at or after RG(n).

Summary. We can now summarize the time of clearance of the n^{th} takeoff as follows:

$$CT(n) = FR(n) \text{ if } RG(n) \leq FR(n)$$
$$= RG(n) + Z(n) \text{ if } RG(n) > FR(n)$$

Finally, we can summarize the delay W(n) to the n^{th} takeoff:

$$W(n) = FR(n) - RG(n) \text{ if } RG(n) \leq FR(n)$$

$$= Z(n) \text{ if } RG(n) > FR(n)$$

6. VARIABILITY OF INTERVALS F(n) AND T(n)

In the preceding formulas we have not specified exactly how variability in the values of the intervals F and T is to be provided for. Such variability can be classified into two kinds:

1. The major effects upon the average values of F and T which are caused by such factors as weather, movement-rate, extremes in type of aircraft population, and runway design and use.
2. For individual successive aircraft in a particular sequence, fluctuations of F and T from the average values for the sequence.

The first kind of variability is essentially parametric, and dependent at most upon position in the sequence of aircraft, that is, upon n. For example, bad weather may be the case throughout a sequence, or only for some portion of it. Similarly for movement-rate, extremes in type of aircraft population, and runway layout and use. Thus this kind of variability is provided for by making the average values of F and T depend simply upon n. If dependence is wanted instead upon clock-time, then an average conversion from n to corresponding time of the clock, and vice versa, may be calculated in the way described in the section below entitled, "Average traffic-process clock-times at CT(n-1)."

The second kind of variability listed above would appear to require great complexity of description. Among the potential sources of fluctuations the following suggest themselves:

1. The obvious effect of type of aircraft pairs on the intervals F and T,
2. The benefits of occasional expediting by pilots upon request,
3. Controllers must estimate the lengths of the intervals G, F, and C in advance,
4. Unexpected fluctuations.

From the standpoint of providing a suitable mathematical model of average delay, the cumulative nature of several fluctuations in succession is the quantity of major concern. In particular, these fluctuations if treated as random variables should not necessarily be assumed to be statistically independent.

Indeed the delay-measurement program revealed that excellent agreement is obtained between observed delay and predicted delay if one assumes that

1. Fluctuations of the first kind are important--that is, the average values of F and T depend on weather, movement-rate, population of aircraft types and runway design and use,
2. For a given sequence of aircraft, the values of F(n) and T(n) should be set at single constants F and T for the entire sequence.

The second of these two findings is perhaps less surprising when one recalls that aircraft traffic flow is not a sharp jerky motion, but rather a flow, the parameters of which flow are always being set a little in advance by decisions and estimates of successive pilots and of controllers. Consequently the delay may for given average flow rates be minimized if the cumulative fluctuations in successive F(n)'s and T(n)'s are such that we may as well treat F(n) and T(n) as if they were simply constant for each n.

This has been done in the equations in the following sections.

7. COMPUTING TAKEOFF DELAY THROUGHOUT SEQUENCE OF TAKEOFFS

The analysis of the delay to the n^{th} takeoff provides a procedure of computing the delay to each takeoff in the sequence in a recursive fashion--that is, in terms of the delay to the previous aircraft. The equation for $W(n)$ can be written as:

$$W(n) = W(n-1) + J(n) - D(n) \text{ if } D(n) < W(n-1) + J(n)$$

$$= Z(n) \text{ if } D(n) > W(n-1) + J(n)$$

This equation may be used for Monte Carlo simulation of the delay process. Either for that purpose, or for direct computation of the probability distribution of $W(n)$, it is necessary to develop the details of the probability distributions of the various intervals composing the delay. The reader unfamiliar with the mathematical methods used may pass over this development.

Average traffic process clock-times at CT(n-1). It should be recalled that from one sample sequence to another, the time CT(n-1) will not always occur the same amount of time after the beginning of the sequence. The fact that the landings and/or takeoff movement rates and the basic intervals of these two processes may be varying with time poses an apparent complication. However, for sequences of substantial length,

this apparent complication may be avoided by approximation methods of suitable accuracy.

Specifically, let $1/g(n)$ be the average length of a gap interval G at the time CT(n-1), and let B(n) be a typical interval B at this same time. We recall that the time CT(n-1) will occur at

$$D(1) + D(2) + \dots + D(n-1) + W(n-1)$$

so that we may find the average value of CT(n-1) by finding the sum of the averages of these several terms. We may then choose the time so obtained as an average point in the landings process so as to obtain the required values g(n), B(n), F(n), and T(n).

<u>Interval K(n).</u> We recall that the interval J(n) is the sum of the two intervals K(n) and H(n). Accordingly, let us first develop each of these intervals separately. Let

$K[T(n);n]$ = probability that $K(n) = T(n)$

$k(x;n)$ = probability density that $K(n) = x > T(n)$

Then we observe that

$$K[T(n);n] = \exp\left\{-g(n)\left[T(n) - F(n-1)\right]\right\}$$

$$k(x;n) = \exp\left\{g(n)F(n-1)\right\}\int_{F(n-1)}^{T(n)} g(n)\exp\left[-g(n)x\right]b(x;n)dx$$

where

$\exp A = e^A$

$b(x;n)$ = probability density that $B(n) = x$

Letting $\underline{k}(\Theta;n)$ be the expected value of $\exp[-\Theta K(n)]$, we can summarize the distribution of $K(n)$ in Laplace transform form as

$$\underline{k}(\Theta;n) = \exp[-\Theta F(n-1)]\{\underline{\ell}(\Theta;n) + [1-\underline{\ell}(\Theta;n)]\exp[-g(n) + \Theta][T(n)-F(n-1)]\}$$

where

$\underline{\ell}(\Theta;n)$ is the expected value of $\exp[-\Theta L(n)]$.

Interval H(n). The interval $H(n)$ is somewhat more complicated. Let

$$\begin{aligned} H(0;n) &= \text{probability that } H(n) = 0 \\ h(x;n) &= \text{probability density that } H(n) = x > 0 \\ g(x;n) &= 0 \text{ if } x > F(n) \\ &= g(n)\exp[-g(n)x] \text{ if } x \leq F(n-1) \\ u(1;x;n) &= \int_0^x g(x-y;n)b(y;n)dy \\ u(k;x;n) &= \int_0^x u(k-1;x-y;n)u(1;y;n)dy \text{ for } k = 2, 3, \ldots \end{aligned}$$

Then an examination of cases will verify that

$$\begin{aligned} H(0;n) &= \exp[-g(n)F(n)] \\ h(x;n) &= \exp[-g(n)F(n)] \sum_{i=1}^{\infty} u(i;x;n) \end{aligned}$$

If we now let $\underline{h}(\Theta;n)$ be the expected value of $\exp[-\Theta H(n)]$, we can summarize the distribution of $H(n)$ in Laplace transform form as

$$\left\{1 - \left[1 - e^{-[g(n) + \Theta F(n)]}\right] \ell(\Theta;n)\right\} \underline{h}(\Theta;n) = e^{-g(n)F(n)}$$

Interval J(n). Let

$j(x;n)$ = probability density that $J(n) = x \geq 0$
$\underline{j}(\Theta;n)$ = expected value of $\exp[-\Theta J(n)]$

Since J(n) is the sum of K(n) and H(n), and since these two terms are statistically independent intervals, then

$$\underline{j}(\Theta;n) = \underline{k}(\Theta;n)\underline{h}(\Theta;n)$$

Interval V(n). This interval is quite simple, namely

$$V(n) = F(n) + B(n) + H(n)$$

Thus, if we let $\underline{v}(\Theta;n)$ be the expected value of $\exp\left[-\Theta V(n)\right]$, then

$$\underline{v}(\Theta;n) = \exp[-\Theta F(n)]\underline{b}(\Theta;n)\underline{h}(\Theta;n)$$

Interval Z(n). We recall that the conditional probability of an interval G* is simply that of the exponential interval G(n). Let

$Z(0;n)$ = probability that $Z(n) = 0$
$z(x;n)$ = probability density that $Z(n) = x > 0$

$$q(1;x;n) = \int_0^x g(x-y;n)\int_0^y v(y-t;n)\exp[-g(n)t]dydt$$

$$q(k;x;n) = \int_0^x g(x-y;n)\int_0^y v(y-t;n)\, q(k-1;t;n)dydt, \quad k = 2, 3, \ldots$$

Then an enumeration of cases will verify that

$$Z(0;n) = \int_0^\infty \lambda(n)\exp[-\lambda(n)t]\{\exp[-g(n)t] + \sum_{i=1}^{\infty} q(i;t;n)\}dt$$

$$= \lambda(n) \div \left[\lambda(n) + g(n) - g(n)\ \underline{v}\ (\lambda(n);n)\right]$$

Moreover, because of the exponential distribution of G(n), we may write

$$z(x;n) = g(n)Z(0;n) \int_x^\infty \lambda(n)\exp[-\lambda(n)(t-x)]v(t;n)dt$$

If we let $\underline{z}(\Theta;n)$ be the expected value of $\exp[-\Theta Z(n)]$, then we can summarize the distribution of Z(n) in Laplace transform form as

$$\left[\Theta - \lambda(n)\right]\underline{z}(\Theta;n) = Z(0;n)\left\{\Theta - \lambda(n) + g(n)\underline{v}(\lambda;n) - g(n)\underline{v}(\Theta;n)\right\}$$

As we shall see, the interval Z(n) is of only passing interest in the delay equations and we omit any derivation of further properties of it.

<u>Probability distribution of W(n) in recursive form.</u>

Let

$W(x;n)$ = probability that $W(n) \le x$
$w(x;n)$ = probability density that $W(n) = x > 0$
$\underline{w}(\Theta;n)$ = expected value of $\exp[-\Theta W(n)]$

Now we may write

$$\text{prob}[D(n) > W(n-1) + J(n)] = \underline{w}[\lambda(n);n-1]\underline{j}[\lambda(n);n]$$

so that

$$W(0;n) = \underline{w}[\lambda(n);n-1]\underline{j}[\lambda(n);n]\ Z(0;n)$$

We may consequently write

$$w(x;n) = W(0;n)z(x;n)/Z(0;n) + \int_x^\infty \lambda(n)\exp[-\lambda(n)(t-x)]\int_0^t j(t-y;n)dW(y;n-1)$$

When these equations are cast into Laplace transform form, and the distribution of the positive part of the interval Z(n) is eliminated, we obtain in summary form:

$$[\Theta - \lambda(n)]\underline{w}(\Theta;n) = W(0;n)\{\Theta + g(n)[1 - \underline{v}(\Theta;n)]\} - \lambda(n)\underline{w}(\Theta;n-1)\underline{j}(\Theta;n)$$

where W(0;n) is given separately. These two equations then summarize the recursion.

Simple as the recursion equations for W(n) appear, it turns out not to be a simple numerical task to carry out the recursive computation. Indeed it appears that, for purposes of getting approximate results, the use of Monte Carlo simulation would be as efficient a means of computation. A properly designed Monte Carlo program would have the further advantage of much greater flexibility in studying the delay process during time periods of exceptional nature (such as severe but very short-lived peaking of the landing or takeoff rates) during which some of the averaging

required to bring the recursion equations to the presented stage of simplicity would not be sufficiently valid.

8. DELAY AS A FUNCTION OF TIME

Now that we have presented the delay by each individual aircraft in the takeoff sequence, it is important to note that there is an alternative delay process--namely, the delay $W(t)$ to a takeoff that becomes ready to go at time t. In contrast to the requirements of averaging that were necessary to keep the delay $W(n)$ reasonably simple of computation, we can develop a differential equation for the process $W(t)$ without having to resort to as much averaging.

For any aircraft becoming ready for takeoff at some time t, let $W(t)$ be its delay--that is, the aircraft will be cleared for takeoff at the time $t + W(t)$. We can view $W(t)$ as a stochastic process and develop it differentially as follows.

If $W(t) > 0$ and no aircraft becomes ready to go in the interval dt following t, then $W(t + dt) = W(t) - dt$. If $W(t) > 0$ and an aircraft does become ready for takeoff in this same interval dt, then $W(t + dt) = W(t) - dt + J$, where J is the interval described earlier.

If, at some time t_o, $W(t)$ becomes 0, then an interval G greater than the interval F required by a takeoff becoming ready to go at t_o is then in progress in the landings sequence. $W(t)$ then continues to remain equal to 0 in value for a length of time equal to $G - F$ (or until time $t_o + G - F$)--unless some aircraft becomes ready for takeoff sooner, in which case $W(t)$ is then increased by the amount J. If $W(t)$ remains 0 in value throughout the time interval $G - F$, then, at time $t_o + G - F$, $W(t)$ is increased by the amount $F + B + H$, which sum we denote by V.

From these considerations, and recalling that the distribution of G - F is exponential (though possibly nonstationary), we can write the following differential equation for

$w(x,t)$, the probability density that $W(t) = x$, namely:

$$\frac{\partial}{\partial t}w(x,t) = \frac{\partial}{\partial t}w(x,t) - \lambda(t)w(x,t) + \lambda(t)\int_0^x w(x-y,t)j(y,t)dy + g(t)W(0,t)v(x,t) + \lambda(t)W(0,t)j(x,t) \qquad (1)$$

where the symbols have meanings as follows:

$W(0,t)$ = probability that $W(t)$ is 0,
$j(x,t)$ = probability density that an interval J which begins at t will have a length of x,
$v(x,t)$ = probability density that an interval V which begins at t will have a length of x,
(t) = takeoff ready to go rate at time t,
$g(t)$ = landing movement rate at time t.

Moreover, for $W(0,t)$, we have the following additional equation:

$$\frac{\partial}{\partial t}W(0,t) = -\left[\lambda(t) + g(t)\right]W(0,t) + w(0,t) \qquad (2)$$

The two differential equations 1 and 2 may be solved numerically by double recursion on x and t, and the resulting values can be tabulated to provide profiles and averages of the distribution of $W(t)$.

Note that, in the formulation expressed in these two differential equations, we are able to incorporate quite well any requirement of nonstationary variation in the move-

ment rates and in the lengths of the various intervals which contribute to delay.

The fact that such solutions were not carried out during the study is justified by the simple fact, that the observations of actual delay, even during relatively short periods of time, showed a remarkable conformity to the values of delay predicted (by finding the average solution of the above equations under the assumption that the probabilities of delay do not change with time, and using average values of the distributions of the various intervals contributing to delay). The delay process so viewed is termed stationary.

We now turn to the solution under stationarity, in the course of which the detailed values of the distributions of the intervals J, V, B, H, etc., will be developed.

9. STATIONARY DELAY

As remarked earlier, actual measurements of delay to departures at a carefully selected variety of U.S. airports produced the result that the observed delays agreed quite closely with the delays computed by assuming the probability distribution of delay to be stationary and using only average movement rates and average interval lengths for the entire period of operation during which the airport was either consistently busy, or consistently slack. This finding strengthened the case for the steady-state solution with the provision that some understanding of the time-dependent case was required to interpret the steady-state answers (see Appendix B).

It is a much easier task to compute the distribution of delay, and particularly its moments, under stationary conditions. Furthermore, both processes--W(n) by aircraft and W(t) by time--yield the same result in the value of delay because of the Poisson nature of the takeoff demand process.

Accordingly we now present the solution to the delay under conditions of stationarity.

When the probability distribution of $W(t)$ and its elements are not changing with time, we may set

$$\frac{\partial W(0;t)}{\partial t} = \frac{\partial w(x;t)}{\partial t} = 0$$

in the differential equations for the $W(t)$ process. We may also suppress the time t in all symbols and quantities. If we do so, the following Laplace transform summarizes the fundamental equation for the delay process W:

$$[\theta - \lambda + \lambda j(\theta)]\underline{w}(\theta) = W(0)\ \{\theta + g[1-v(\theta)]\}$$

From this equation we may, by identifying coefficients of θ^k, find

$W(0)$ = the probability that the delay W is 0

w_n = the average value of W^n

(The variance of W is just $w_2 - w_1^2$.) In particular, we find that

$$W(0) = \frac{1 - \lambda j_1}{1 + gv_1}$$

$$w_1 = \frac{\lambda j_2}{2(1 - \lambda j_1)} + \frac{gv_2}{2(1 + gv_1)}$$

where j_n is the average value of J^n and v_n is the average value of V^n.

We give these results in the following section in a form suitable for computation.

From the formula for W(0), we observe that the maximum average takeoff demand rate that can be handled under stationary conditions is

$$\lambda = \frac{1}{j_1}$$

It should be forcefully emphasized however that, at this rate the probability distribution of delay is unstable in time and indeed delays now tend to become systematically larger and larger for successive aircraft. Consequently, only takeoff demand rates substantially below this maximum can be handled in actual practice. This fact may be verified by noting that, in the first of the two terms composing the average delay, the denominator is proportional to the fraction by which the actual traffic load (expressed in aircraft per unit time multiplied by time per unit aircraft) is less than the maximum value of 1.

10. DETAILED FORMULAS FOR COMPUTING WAIT

Following our usual notational practice, we denote the expected values respectively of L^1, B^1, V^1, H^1, K^1, and J^1 by ℓ_1, b_1, v_1, h_1, k_1, and j_1. Now it is readily verified that

$$\ell_1 = \frac{1}{g} + b_1$$

$$\frac{\ell_2}{2} = \frac{b_2}{2} + \frac{\ell_1}{g}$$

(When R and C are taken as constants, then $v^2(B) = 0$ and $b_2 = B^2$, $B = R + C$).

Further, it is not hard to show from the equations for $\underline{v}$ (Θ) and $\underline{h}$ (Θ) that

$$1 + gv_1 = g\, \ell_1 e^{gF}$$

and

$$\frac{v_2}{2} = e^{gF} \left(h_1\, \ell_1 + \frac{b_2}{2}\right)$$

so that

$$\frac{gv_2}{2(1+gv_1)} = h_1 + \frac{b_2}{2\, \ell_1}$$

of course

$$\ell_1 = \frac{3600}{\lambda_L}$$

Consequently w_1 reduces to

$$w_1 = h_1 + \frac{\lambda_T\, j_2}{2(1-\lambda_T j_1)} + \frac{b_2}{2\, \ell_1}$$

which is exactly the same equation (note changes in notations) given for the average wait in the first report,

reference 1 on the SAM model when B, F, and T are constants.

For convenience we record here the component terms of w_1 in the above equation:

$$h_1 = \ell_1 \; (e^{gF} - 1) - F$$

$$j_1 = \ell_1 \; e^{gF} \; (1 - e^{-gT})$$

$$\frac{j_2}{2} = j_1 h_1 + e^{gF} \left[\frac{\ell_2}{2} \; (1 - 3^{-gT}) - \ell_1 \; T \; e^{-gT} \right]$$

11. DELAYS AT SINGLE ISOLATED RUNWAY USED ONLY FOR TAKEOFFS

We may obtain the analysis of the case of an isolated runway used only for takeoffs by simply removing the landings process from the analysis of such a runway used for both landings and takeoffs. We first set all landing rates g equal to 0 and set $L(\theta)$ and $a(\theta)$ equal to 0. This removes the landings process. As a consequence we note that

interval K = T
interval H = 0

Therefore,

interval J = T
intervals Z and V = 0

As a result, the equations relating W(n) and W(n-1) become

$$W(n) = W(n-1) + T(n) - D(n) \text{ if } D(n) < W(n-1) + T(n)$$

$$= 0 \text{ if } D(n) \geq W(n-1) + T(n)$$

However, this simplification does not remove the inherent numerical problems in computing the distribution of W(n) recursively, unless the intervals T(n) are all constant.

The differential equations for the probability distribution of the wait W(t) viewed as a function of time now become

$$W(0;t) = -\lambda(t)W(0;t) + w(0;t)$$

and

$$\frac{\partial}{\partial t} w(x;t) = \frac{\partial}{\partial x} w(x;t) - \lambda(t)w(x;t) + \lambda(t)\int_0^x \tau(x-y;t)dW(y;t)$$

where $\tau(x;t)$ is the probability density that an interval T which begins at time t has a length x. The stationary solution was also found by direct observation to produce observed delays quite in agreement with the theoretical. The Laplace transform of the stationary solution is in this case

$$\left[\theta - \lambda + \lambda t(\theta)\right] w(\theta) = W(0)$$

From this, we find that

$$W(0) = 1 - \rho$$

where

$$\rho = \lambda t_1$$

We also find that

$$w_1 = \frac{\lambda t_2}{2(1 - \lambda t_1)}.$$

In this case, the saturating takeoff demand rate is just $1/t_1$. The same considerations discussed before warn against loading the runway at this rate for a considerable period of time.

12. DELAYS TO TAKEOFFS IN INTERSECTING RUNWAY OPERATIONS

As described in Sections II, III, IV, and VI, the SAM model with properly interpreted inputs is valid for such configurations but some additional background is presented here relevant to the actual mathematics that supports the interpretation of inputs.

Intersecting runway designs (that is, designs in which one or more of the runways to be used intersect) possess advantages of land utilization and of cross-wind accommodation, and can affect the terminal-to-runway taxi time of aircraft. They can also increase air-ground exchange capacity over a single runway. This capacity, and the associated delays to aircraft, is our primary concern here. As we shall see, the amount of increase in capacity or, correspondingly, of reduction in delay for a given movement rate, provided by two or more runways above that of a single isolated runway depends very much upon the location of the point(s) of intersection of the runways. For example, if the point(s) of intersection are located quite far from the takeoff (and touchdown) ends of the runways, little if any advantage is produced over a design using one single runway in place of all the runways.

From the viewpoint of obtaining a mathematical analysis of delay to aircraft at the runways, cross-runway operation introduces three new considerations not explicitly pres-

ent in the analysis of single-runway operation and in the formulas developed for it. These are:

1. The order in which the runways are used by the successive aircraft of a sequence.
2. The minimal time separations between two successive aircraft depend upon which runways are being used by the aircraft.
3. A landing or takeoff on one runway blocks that runway and any intersecting runway for certain associated time intervals.

It is possible to present a delay analysis that incorporates all three of these considerations explicitly. As we shall see, it is not possible to derive from the analysis formulas for delay which are as simple as those for a single-runway design, even under stationary conditions. However, in most practical cases, there is a way of properly adapting the delay formulas for single-runway operation so as to yield close approximations of the delay in cross-runway operation. The accuracy of these approximations have been verified against actually observed delays of aircraft in cross-runway operation.

It is interesting to note that just as the three considerations complicate the mathematical analysis of delay in cross-runway design, so also do they complicate the task of the airport controller in his endeavor to move aircraft as expeditiously as possible and thereby minimize delay. Accordingly, let us examine them more closely, first with respect to takeoffs.

Let the number of intersecting runways to be used be N (normally N = 2 or 3). Then some portion λ_i of the total takeoff movement rate λ will use runway i. Thus,

λ_i = average takeoff demand rate for runway i

so that

$$\lambda = \sum_{i=1}^{N} \lambda_i$$

Of more direct usefulness in the delay analysis is the ratio

f_i = fraction of takeoffs using runway i = $\frac{\lambda_i}{\lambda}$

For takeoffs, we must decide the rule to be used to determine on which runway the n^{th} takeoff of a sequence will occur given that the n-1st takeoff occurred on some specified one of the runways.

In actual practice this choice of runway is sometimes a difficult one between two conflicting attractions: (1) the reduction in inter-takeoff separation times achievable from making a systematic alternation of runways among successive aircraft in the takeoff sequence, and (2) the basic policy requirement of first-come first-served, which refers to the order in which aircraft are logged by the controller as they report ready to go, which order is in time and is separate from consideration of the runway to be used.

The advantage of alternation occurs when

T_{ij} = minimum safe separation time interval between two successive takeoffs, the first of which uses runway i and the second runway j.

is less then T_{ii} and T_{jj} when $i \neq j$.

The advantage of systematic alternation could be had within the first-come first-served policy if pilots could accept any runway assignment and if the controller could predict the ready-to-go times of departing aircraft at the times they first enter the taxiway complex runway bound. However, a number of factors act to prevent maximum advantage being taken of extremely systematic alternation.

In the first place, pilots may insist upon the runway of their choice, and such choice is for example the more justified under the very wind conditions that make use of multiple runways meteorologically feasible or even mandatory. Second, the unpredictability of engine warmup times for piston aircraft may frustrate an attempt to load the runways in advance with the aircraft required to achieve a complete alternation. Third, post-takeoff separation requirements between aircraft that use different runways but will use the same departure fix can interrupt a planned alternation. Fourth, ground congestion also acts, particularly under heavy delays, to frustrate complete alternation efficiency just in those designs which might appear to offer most advantage. For example, takeoff-bound aircraft must cross an inner runway to reach an outer runway, and takeoff queues must be stored at points of ready access. Finally, the advantage of systematic alternation is substantially inhibited by the fact that, when landings are included, the quantitative utilization of the runways by takeoffs alone is in itself comparatively light. Consequently, choosing the runway so as to obtain a minimal interval T is not always as important as being able to choose it so as to obtain minimal intervals of F and R.

In view of all these considerations, it seemed advisable in the study to test experimentally the reliability of the assumption that the assignment of successive aircraft to runways should be assumed to be random within the average frequencies f_i.

Note that this does not mean that the advantages of alternation are supposed never to be obtained; rather the frequency of advantage is assumed to be average. The degree of agreement between predicted and actually observed delays was sufficient to support the retention of this assumption.

In mathematical terms, this means that the runway allocation rule for takeoffs is taken to be as follows. If a takeoff occurs on runway i, then for any runway j the probability that the next takeoff occurs on runway j is simply f_j.

13. TAKEOFF DELAYS WHEN NO LANDINGS USE RUNWAYS

To present some of the aspects of the more complicated analysis required by multiple-runway operation, the case of a system of runways (and an associated airspace) used solely by takeoffs will be described. The delay which a given takeoff incurs now depends in general upon which runway it uses. Accordingly, we let $W_i(t)$ denote its delay if it becomes ready to go at time t and uses runway i. At any given time we can represent the delay situation by the list of delays $W_1(t), W_2(t), \ldots, W_N(t)$. Although the individual delays in this list are in general different in value from each other, nevertheless we recall that they are all of the same order of magnitude under a first-come first-served discipline.

Consider now a particular one of these delays, say $W_j(t)$, and consider what happens in a time interval t to t + dt. If no additional aircraft becomes ready to go in dt, then

$$W_j(t + dt) = \begin{cases} W_j(t) - dt \text{ if } W_j(t) > 0 \\ 0 \text{ if } W_j(t) = 0 \end{cases} \qquad (1)$$

If, however, a takeoff does become ready to go in the interval dt on some runway i then it will be cleared to takeoff at time $t + W_i(t)$. The next takeoff on runway j could not then occur until $t + W_i(t) + T_{ij}$, or at the time $t + W_j(t)$, whichever is the later.

Thus, in this case,

$$W_j(t + dt) = \max\left[W_j(t) - dt,\ W_i(t) - dt + T_{ij},\ 0\right] \qquad (2)$$

To transform the list of delays at time t into the list at time t + dt, we must transform each member of the list simultaneously for each possible choice of runway i by equation 1 if no takeoff becomes ready to go in the interval. If one does become ready to go, we must transform each member of the list simultaneously by equation 2 for each possible choice of i. For Monte Carlo purposes, the transformation is quite easy to achieve, but for analytic purposes, the transformation is represented by a quite complicated set of probability equations. Fortunately, the transformation becomes much simpler if we consider the details of the relationships between the magnitudes of the minimal separations T_{ij} in ordinary operation. For it turns out that we need consider only two basic classes of runway designs (or more precisely, design-operation combinations). We denote these two classes by the terms far intersections and near intersections, respectively. Specifically, far intersections means that for any i, j, and k, we can assume that $T_{ik} \leq T_{ij} + T_{jk}$. The inequality here will be referred to as the triangle inequality. Near intersections means that, for some i, j, and k, the above triangle inequality fails to hold often enough to be able to assume that it never holds for that choice of i, j, and k.

Far intersections. In this case, for three successive operations on any runways i, j, and k (not necessarily different), we can assume that $T_{ik} \leq T_{ij} + T_{jk}$. As a consequence, we can assume that for any i and j,

$$W_j(t) \leq W_i(t) + T_{ij} \qquad (3)$$

To see that this is so, we note first that it is certainly true when $W_i(t)$ and $W_j(t)$ are both 0 (as will sometimes be the case). Now suppose it is true at some time t. If no takeoff becomes ready to go in the interval dt following t, then it remains true at time t + dt. Suppose instead that a takeoff does become ready to go during dt, and on runway k. Then since

$$W_j(t + dt) = \max\left[W_j(t) - dt,\ W_k(t) - dt + T_{kj},\ 0\right],$$

$$W_j(t + dt) = W_k(t) + T_{kj} - dt \qquad (4)$$

Similarly,

$$W_i(t + dt) = W_k(t) + T_{ki} - dt$$

Consequently,

$$W_j(t + dt) = W_i(t + dt) + T_{kj} - T_{ki}$$

$$\leq W_i(t + dt) + T_{ij}$$

since, by the triangle inequality, we can assume that $T_{kj} \leq T_{ki} + T_{ij}$. Thus, equation 3 is always true. Moreover, equation 4 shows that, whenever a takeoff occurs on some runway k, then every $W_j(t)$ is reset (actually increased) to the value $W_k(t) + T_{kj}$. In the intervals between the becoming ready to go of takeoffs, all the $W_j(t)$ change independently of each other.

Before showing how these facts permit a fairly simple analytic solution of the delay for each runway, we discuss near intersections.

Near intersections. In this case, for some (possibly all) runways i and k, there is some runway j for which

$$T_{ij} + T_{jk} < T_{ik}$$

Ordinarily this occurs when the intersection of runways i and j is near their starting ends, and when the intersection of j and k is also near their starting ends, but when the intersection of runways i and k is quite far from their starting ends. Now while in very light winds, three different runways i, j, and k may be involved, the most frequent form of the occurrence is when i and k are the same runways, and we confine our discussion to that supposition.

Accordingly, we shall say that i and j are near if

$$T_{ij} + T_{ji} < T_{ii}$$

and for convenience we shall say that a runway i is far from itself. For two such runways i and j, our problem is that a minimal separation T_{ij} cannot be followed by a minimal separation T_{ji}, but at the least must be followed by $T_{ii} - T_{ij}$. Accordingly, for i and j near, we need to distinguish two kinds of minimal separations:

$$_0T_{ij} = T_{ij}$$

$$_1T_{ij} = T_{jj} - T_{ji}$$

14. ANALYSIS OF DELAY

Let

$W_{ij}(x)$ = probability that the delay of a takeoff is $\leq x$ if it uses runway j and follows a takeoff on runway i.

If i and j are near, we separate $W_{ij}(x)$ into two mutually exclusive parts:

${}_0W_{ij}(x)$ when the minimal separation can be T_{ij}

${}_1W_{ij}(x)$ when the minimal separation must be $T_{jj} - T_{ji}$.

An examination of cases will verify that the delay equations aircraft by aircraft through a takeoff sequence are:

i, j far: $$W_{ij}(0) = \sum_k f_k \, \underline{w}_{ki}(\lambda) \, \underline{t}_{ij}(\lambda)$$

i, j near
$$\begin{cases} {}_0W_{ij}(0) = \left[\sum_{k\neq j} f_k \, \underline{w}_{ki}(\lambda) + f_j \; {}_1\underline{w}_{ji}(\lambda)\right] \underline{t}_{ij}(\lambda) \\ {}_1W_{ij}(0) = f_j \; {}_0\underline{w}_{ji}(\lambda) \, {}_1\underline{t}_{ij}(\lambda) \end{cases}$$

Following our standard notational practice, let ${}_0w_{ij}(x)$, ${}_1w_{ij}(x)$, and $w_{ij}(x)$ be, for $x > 0$, the probability densities corresponding respectively to ${}_0W_{ij}(x)$, ${}_1W_{ij}(x)$ and $W_{ij}(x)$. Then

i, j far $$w_{ij}(x) = \int_0^\infty \lambda e^{-\lambda t} \int_0^{t+x} t_{ij}(y) \sum_k f_k \, dW_{ki}\,(t + x - y)dt$$

i, j near
$$\begin{cases} {}_0w_{ij}(x) = \int_0^\infty \lambda e^{-\lambda t} \int_0^{t+x} t_{ij}(y) \left[\sum_{k\neq j} f_k \, d\, W_{ki}(t + x - y) + \right. \\ \left. \qquad f_j \, d_1W_{ji}\,(t + x - y)\right] dt \end{cases}$$

$$ {}_1w_{ij}(x) = \int_0^{\infty} \lambda e^{-\lambda t} f_j \int_0^{t+x} {}_1t_{ij}(y)\, dW_{ji}(t + x - y) $$

The summary representation in Laplace transforms of these equations is:

i, j far: $(\theta - \lambda)\, \underline{w}_{ij}(\theta) = \theta W_{ij}(0) - \lambda \underline{t}_{ij}(\theta) \sum_k f_k\, \underline{w}_{ki}(\theta)$

i, j near
$$ \begin{cases} (\theta - \lambda)\ {}_0w_{ij}(\theta) = \theta\, {}_0W_{ij}(0) - \lambda \underline{t}_{ij}(\theta) \left[\sum_{k \neq j} f_k\, \underline{w}_{ki}(\theta) + f_j\ {}_1\underline{w}_{ji}(\theta) \right] \\ (\theta - \lambda)\ {}_1w_{ij}(\theta) = \theta\, {}_1W_{ij}(0) - \lambda f_j\ {}_1\underline{t}_{ij}(\theta)\ {}_0\underline{w}_{ji}(\theta) \end{cases} $$

These equations may be solved simultaneously for the $w_{ij}(\theta)$, the values of the $W_{ij}(0)$ being found from the roots of the determinant of the equation set.

However, a simple approximation to the solution may be obtained. As a preface to this approximation, the following properties of the above indicated simultaneous solution are noted.

By identifying the coefficients of θ^0 and θ in the equations for i and j near, we find that if a takeoff on runway j follows a takeoff on runway i, then the probability

$$ {}_0W_{ij0} = \text{probability it can be separated by } T_{ij} = \frac{1 - f_j}{1 - f_i f_j} $$

and

$$ {}_1W_{ij1} = \text{probability it must be separated by } {}_1T_{ij} = T_{jj} - T_{ji} = \frac{f_j^2}{1 - f_i f_j} $$

Moreover, the probability of no delay is

$$W(0) = \sum_{i,j} f_i f_j \, W_{ij}(0) = 1 - \rho$$

where

$$\rho = \sum_{i,j} \rho_{ij} \text{ and}$$

for i near j,

$$\rho_{ij} = \frac{\lambda f_i f_j}{1 - f_i f_j} \left[(1 - f_j)\, t_{ij1} + f_j^{\,2} (t_{jj1} - t_{ji1}) \right]$$

for i far from j,

$$\rho_{ij} = \lambda f_i f_j \, t_{ij1}$$

Although the mathematical basis has been developed, it was found and validated that it was more convenient to adjust the values of T for near intersections rather than adding a new equation to the delay formulas. Thus, the final interval T used in the computer program is protected by suitable adjustments for near intersections. Also, the interval of departure/arrival/departure on intersecting runways must be protected to ensure that the departure to departure intervals are not violated. An illustration of this type of adjustment follows in Chapter VI, Section A.

15. "SINGLE-RUNWAY MODEL" APPROXIMATION TO CROSS-RUNWAY OPERATION

When the average delay is not too small, then the delay to any takeoff is approximately independent of the choice of runway, since most of the delay is simply waiting

for the aircraft ahead to be cleared for takeoff and the separation from that aircraft is but a small part of the delay. That is

$$W_{ij}(x) \approx W(x) = \text{wait of an average takeoff.}$$

If this approximation is used, then we obtain a single equation for the delay--namely, the single-runway model equation

$$\left[\theta - \lambda + \lambda\ \underline{t}(\theta)\right]\ \underline{W}(\theta) = \theta\ W(0)$$

where

$$\underline{t}(\theta) = \sum_{i,j} \underline{t}_{ij}(\theta)$$

and

$$\text{for } i \text{ near } j,\quad \underline{t}_{ij}(\theta) = \frac{f_i f_j}{1 - f_i f_j}\left[(1 - f_j)\,{}_0\underline{t}_{ij}(\theta) + f_j^{\,2}\,{}_1\underline{t}_{ij}(\theta)\right]$$

$$\text{for } i \text{ far from } j,\quad \underline{t}_{ij}(\theta) = f_i f_j\ {}_0\underline{t}_{ij}(\theta)$$

Thus we have merely an average separation T, the average being taken over all possible pairs i and j of runways using the proper frequencies of occurrence of the separations ${}_0T_{ij}$ and ${}_1T_{ij}$ if i and j are near.

VI. PREPARATION OF AIRPORT CAPACITY HANDBOOK

A. GENERAL

In analyzing a specific airport design, the prime requirement is to calculate aircraft delay versus movement rate. Having chosen an average delay at which the airport is considered to be at capacity operation, it is then possible to pick out the movement rate for that average delay.

Basically, the technique can be summarized as follows:

1. Describe the layout of the airport.
2. Describe the nature of the traffic demand.
3. Determine the runway configurations and their use.
4. Calculate the model inputs knowing the runway configurations, runway use, and nature of the demand.
5. Insert the inputs into SAM and FIM and calculate the delays versus the movement rates.

The development of a single delay curve is complicated by the fact that many of the inputs change value as a result of the pressure factor as the movement rate increases. Also, for some runway configurations there are quite complex relationships between the inputs.

The SAM model itself is not a simple equation. In the previous report, a number of delay curves were produced for single runways with various populations, and a few for intersecting runways. Although the SAM model itself was calculated on a computer (LGP-30), it took about a minute to calculate each point on the curve. This was rather slow and the inputs to SAM also had to be calculated entirely by hand.

For the Airport Capacity Handbook, it was estimated that a large number of figures would have to be produced that

would involve a great number of individual cases. Therefore, some method was required to automate the input preparation and to speed up the model calculations. Thus, it was decided to write a program for the IBM 7090 computer that would fulfil the requirements.

The hand-calculation of SAM/FIM inputs for any runway configuration is a tedious process involving the determination of inputs as averages weighted by the probabilities of aircraft sequence for each movement rate.

In addition, for intersecting runway configurations, there are some rather complex limits and checks on some of the inputs. To illustrate two of these limiting factors, a configuration where the intersections are close to the runway thresholds provides an excellent example.

Figure 6-1 shows such a configuration where the runway use is such that landings are on runway 1 and takeoffs are on runways 1 and 2.

If a departure takes off on runway 1 and then is followed by a departure on runway 2, the interval T is the time from clear to takeoff to passing through the intersection of the departure on runway 1. If the runway 1 departure is a jet and that on runway 2 is a small light aircraft, there is a possibility that if the light aircraft were released for takeoff immediately after the jet passes through the intersection, the light aircraft would encounter very bad turbulence on reaching the intersection. Therefore, at such airports the controllers could be expected to hold some departures for clearance until they were satisfied that no danger existed.

At no airports visited during the field surveys was such a configuration observed. However, in planning the Airport Capacity Handbook, it had to be assumed that such a con-

figuration might well exist elsewhere. Therefore, to cover such cases, some estimates had to be made of the minimum times for such intersection crossings. The field surveys taken where no such conditions were seen on close intersections provided some basis for the limits, and jet wake velocity data from reference 6 gave some indication of the distances behind jet aircraft which had to be protected.

Thus, the protection behind a Class A aircraft on takeoff followed by a Class E was estimated at 22 seconds minimum.

In Figure 6-1, if a Class A aircraft takes off on runway 1 and is followed by a Class E on runway 2, and if the clear to takeoff to intersection for the Class E is less than 22 seconds, the value of T must be set at 22 seconds. These corrections to T must be calculated for each class sequence and each correction must be weighted by the probability of each sequence and finally applied to the average value of T.

A second example of a limiting factor on intersecting runways is the sequence (Figure 6-1) of a takeoff on runway 1 followed by a landing on runway 2 followed by a takeoff on runway 1.

In terms of the SAM inputs, this is F + C + R, where F is the time for the first takeoff preceding the landing. The landing aircraft takes up the interval C and then R (over threshold to intersection). On the completion of the interval R, the second takeoff may be released. However, it must be ensured that the interval $F + C + R \geq T$, where T is the interval between two successive takeoffs on the same runway. If this condition were not held, the model would allow successive takeoffs too close to each other, thus violating the rules for successive takeoffs on the same runway.

Thus, if F = 20 seconds, C = 6 seconds, R = 13 seconds, and T = 55 seconds, there would be a correction of T - (F + C + R) = 16 seconds.

If the probability of this sequence were 0.25, the final input correction on the average T value (SAM input) would be 16 x 0.25 = 4 seconds.

The basic flow of the program in its completed state is as follows.

1. Manually insert a description of the airport comprising the following items:
   (a) VFR or IFR
   (b) Configuration, including number of runways.
   (c) Population of arrivals and departures by class for each runway.
   (d) Ratio of arrivals to departures.
   (e) Runway lengths and intersection distances.
   (f) Runway rating for each arrival runway.
   (g) Number of departure fixes (IFR only).
   (h) Percentage of departures by runway using each departure fix (IFR only).
   (i) Angle between runways (IFR only)
   (j) Longitudinal distance between thresholds (IFR, close parallels only).
   (k) Year (1963 or 1970).
2. The computer then automatically sets itself at a starting value for λ_S (total movement rate) on the basis that in VFR λ_L or λ_T cannot be less than 10. The ratio of arrivals to departures then establishes both λ_L and λ_T. In IFR, the limit for λ_L or λ_T is 5.
3. The computer then calculates R, C, T, and F using the stored data (in the form of tables and equation of curves) preset conditions, and manual input.

4. The departure delay is then calculated by the computer using the above inputs (plus λ_L and λ_T) in the SAM subroutine.

5. a_1 and a_2 are then calculated and entered in the FIM subroutine with λ_L to calculate arrival delay.

6. The computer must now reset and recalculate SAM and FIM at a higher movement rate. It is programmed to increase λ_S at a rate dependent upon SAM or FIM delay. If the delays are low it will take a larger increase of λ_S from the starting value. As it works through increasing values of λ_S the delays will begin to increase and to ensure accuracy, the computer will increase λ_S in correspondingly smaller increments.

7. Eventually SAM will give a result of infinite delay. If FIM has not yet reached this stage, the computer will continue increasing λ_L until FIM reaches infinite delay. However, when SAM has reached infinite delay, the computer switches to another mode of operation in additon to calculating FIM.

8. The service times R + C and F are multiplied by their respective movement rates (λ_L and λ_T) to calculate utilization on the basis of perfect alternating nonrandom arrivals and departures. When $\lambda_L(R + C) + \lambda_T(F) = 3600$, then utilization is 100 percent. If at this stage FIM has reached infinite delay, the computer will stop and go on to the next run. If FIM has not been completed, the computer will continue until FIM is at infinite delay.

9. At this stage and before going on to the next run, the computer is programmed to automatically compute capacity. Arrival and departure capacity are both governed by average delay for a given movement rate. For example, in most cases a departure delay of 240 seconds dictates λ_S capacity. Since 240 seconds delay may occur at some intermediate value, the computer must interpolate for the appropriate λ_S.

10. On completion of this complete cycle, the results (output) are stored on magnetic tape for printing and the program is reset for the next case.

A flow diagram of this operation is shown at Figure 6-2. An example of the final output in printed form is shown in Figure 6-3 together with explanatory notes.

To summarize, the characteristics of the program are:

1. An IBM 7090 Fortran program (binary deck, 700 cards).
2. Running time, 25 cases per minute. Depending upon manual input, can solve VFR, IFR 1963, or IFR 1970 for the following:
   (a) Single runway, mixed operations, landings only or takeoffs only.
   (b) Intersecting runways (and open V configurations, operations towards the apex). Up to six runways in VFR with any combination of runway use, up to three runways in IFR with landings restricted to one runway.
   (c) Close parallel runways (two) in IFR.

All curves of time versus movement rate for inputs T, F, R, and A are stored as formulas, and constant values for C, F (IFR), etc. are stored in table form. Therefore, it is a fairly simple process to update or change any such values. This is particularly valuable when new field data becomes available since it can readily be entered in storage, replacing the old values.

B. HANDBOOK DESCRIPTION

In arriving at the final layout of the Airport Capacity Handbook several distinct steps had to be taken. The first major step was to discover, for each runway configuration, what parameters had the greatest effect on capacity. Therefore, a series of test cases was assembled and computed.

From these test cases, it was established that for single runways in VFR, population had a very great effect on capacity together with runway rating. For two intersecting runways in VFR, the population had a lesser effect, but position of the runway intersections was very critical.

This testing was done for all the configurations necessary for analyzing airport design. From these test cases, it became apparent that:

1. To obtain accurate predictions of airport capacity, many more cases would have to be run than was originally proposed. This was particularly true of intersecting runways with various combinations of runway use.
2. There could be no common system presented for computing capacity for all the combinations of runway configurations and usage since each was affected by different paramters.
3. Because of the many different designs at existing airports around the country, a simple pictorial display of the runway with capacities representing various populations was not possible.
4. Because of items 2 and 3, it was felt that a simplified graphical technique covering each basic type of configuration would be more meaningful and would serve to educate the user in some of the subtler aspects of airport capacity.

Therefore, a technique was developed where the population was broken down into groups determined by the initial testing. For each population group, an average of all or some of the parameters was chosen and a delay/operating rate curve determined. To enclose the variation of some of the parameters, correction factors were then calculated and presented in graphical form.

A good example of this technique is to show the procedure necessary for two intersecting runways in VFR. First, an initial assumption was made that the ratio of arrivals to departures was 1.0 and that runway use was arrivals on one runway only and departures on the other. From the initial testing, 15 population groups were chosen. For each of these groups, typical runway lengths were chosen. Since total run-

way occupancy has a small effect on the capacity of intersecting runways, an "average" runway rating was calculated for each group.

For the basic delay/operating rate curve for each group the runway configuration was set up so that the intersection distances were half of the respective runway lengths. To obtain either capacity or a delay/operating rate curve for a configuration other than the basic one, a series of configurations where runway intersection distances were varied were run on the computer and their capacities compared with that of the basic one. The correction factors thus obtained were then plotted in graphical form. The varying parameters were thus the relationships of the intersection distances of each runway to the runway lengths chosen.

Thus, on one figure the user can determine capacity and/or a delay curve for any two-runway intersecting configuration for a given population grouping. For alternative runway uses and varying arrival-to-departure ratios, a separate figure-of-correction factor had to be calculated for each.

Since the correction curves for many of these parameters are rather complex, the number of cases that had to be run was very large, and to complete the Handbook over 4000 cases were completed.

Before completing the description of the Handbook there are three aspects that require some additional notes.

1. DETERMINATION OF DELAY LEVELS FOR CALCULATING CAPACITY

In the previous report the average delay at which an airport was considered to be at capacity was 6 minutes average departure delay. Since only VFR was considered, arrival delay was generally of little significance.

However, the Handbook required determination of capacities for many different types of airport configurations involving varied populations of aircraft, with many combinations of runway use under VFR and IFR conditions, including arrivals only.

Also, it was necessary to relate the steady-state solutions to nonstationary demands of aircraft. Appendix A deals with the latter aspects and the rules of analysis are further described in Chapter 4 of the Airport Capacity Handbook, under Capacity versus Demand.

In choosing the actual delays of 4, 3, or 2 minutes as described in Appendix B, the main criteria was one of queue length, although other items such as safety and the cost of delay were considered.

Where an airport is handling air-carrier traffic (Classes A and B), the practical limit of average departure was chosen at 4 minutes. The reason for choosing 4 minutes rather than 6 minutes was that the airport surveys indicated that, as departure delays approached or reached 4 minutes average, some aircraft were delayed for as long as 20 minutes and queue lengths were becoming excessive. The highest delays recorded were at LaGuardia where the average delay on one day was over 4 minutes and many pilots were heard to complain of their delays.

On the basis of a 4-minute delay, the average queue length can be calculated if the service times are known. Where any appreciable numbers of air-carrier aircraft are present, service times (that is, separation times) will vary between 40 and 80 seconds in VFR. Queue length is obtained from the equation:

$$\text{Queue length} = \frac{\text{Average delay}}{\text{Average service time}}$$

Assuming an average service time of 60 seconds, this gives an average queue length of four aircraft. Examining the distribution of delay for a 4-minute average, it is found that maximum delays of 18 minutes are possible for less than 1 percent of the aircraft. This results in the fact that queue lengths can go up to a maximum of 18, less than 1 percent of the time.

However for 6-minute average delays, maximum delays of 30 minutes can be expected, which would give maximum queue lengths of 30 aircraft. This is obviously an excessive amount and would not be practical, and for this reason the 4-minute average delay is considered a more reasonable figure.

The effect on non-air-carrier airports (no Class A or B aircraft) is interesting. Service times are now reduced to anywhere from 20 to 40 seconds. Assuming 30 seconds as the average, 4-minute average delays would result in queue lengths averaging eight aircraft and maximums of 36. This is obviously much too high, whereas an average delay of 2 minutes gives an average queue length of four and a maximum of 18. For this reason an average delay of 2 minutes represents a practical capacity for such an airport.

This aspect of delay and queue length is also of interest when applied to the arrival situation in VFR.

In VFR where pilots are sequencing themselves, the only way arrival delay can be absorbed is in the air-traffic pattern around the airport. If a 4-minute average arrival delay were accepted it would mean that a maximum of 18 aircraft would have to be accepted in the traffic pattern at times. To absorb the maximum delays of 18 minutes some aircraft would have to stretch their downwind legs to 9 minutes, which would then result in 9 minutes extra on finals to total 18 minutes delay. Since aircraft are traveling at speeds of 2 to 2-1/2 miles per minute (120 to 150 knots), this would

involve path stretching downwind of up to 18 to 27 miles. This is obviously unacceptable.

For this reason, arrival delay for practical VFR capacity of an air-carrier airport is determined from an average arrival delay of 1 minute. This results in maximum delays of about 4 to 5 minutes and maximum path stretching downwind in the order of 4 to 6 miles. This is obviously a more realistic practical limit.

However, at a general-aviation airport aircraft speeds are much slower, between 1 and 2 miles a minute (60 to 120 knots). A 2-minute average arrival delay will result in maximum queue lengths in the order of 18 aircraft which certainly is a limit, but a practical one since small aircraft can occupy relatively small amounts of airspace. Maximum delays of 9 minutes or 4-1/2 minutes downwind stretching would have to be accommodated. This would result in a 7-mile maximum path stretch at an average speed of 1-1/2 miles per minute.

Therefore, it is quite apparent that departure and arrival capacities in VFR based on delay depend upon aircraft population. This concept was used in the preparation of the Airport Capacity Handbook and the computer program was set up on this basis.

In IFR, two things change: (1) stacking of arrivals is possible, and (2) service times increase, particularly where the general-aviation aircraft are concerned. Therefore, an average delay of 4 minutes for both arrivals and departures can be used to determine capacity in IFR regardless of population.

2. <u>IFR OPERATIONS IN VFR WEATHER</u>

At some of the larger air-carrier airports, operations are sometimes conducted under IFR rules of operation

when the weather is VFR. This applied particularly to arrivals and usually occurs in marginal weather conditions--for example, 5 miles visibility and 3000 feet cloud base.

Where there is more than one initial departure routing, departures seem to be not too greatly affected by such operations. Also the effect on arrivals is not identical to IFR since VFR flights are often intermingled with the IFR arrivals. Also, depending upon airport runway configuration, the arrivals may use an ILS approach to one runway but break off at some 5 miles to use other landing runways, thus following a normal VFR operation.

For the purposes of this analysis and use of the Handbook, the capacities and/or delays computed for VFR can be used under these conditions.

It should be mentioned here that a project is presently under way to examine the effects of weather on airport capacity, and it is hoped that some definite conclusions will be reached on the IFR operations in VFR (Project FAA/ARDS-605).

3. AIRPORT OPERATIONS IN 1970

For long-term airport planning, it was considered essential to provide some method of calculating airport capacity in the period 1970-75.

With regard to VFR operations, the airport surveys have made it clear that present-day operational practices result in maximum use of available runways provided their use is not overly restricted by noise abatement regulations. Although some aids should be made available to ease the controller's workload, it is not expected that any great increases in airport capacity can be made. The only exception to this would be in the area of some automation of taxiway-runway intersection crossings.

However, in IFR it is likely that improved ground equipment and techniques could increase present operating rates considerably. In computing capacity or delays for this period, it was necessary to introduce new model inputs that could be used for forecasting. Existing data was examined very closely and some estimates made. These will be examined briefly here.

SAM

1. Neither λ_L nor λ_T change by definition.
2. Changes in C would tend to be rather small and would have little effect on capacity. Thus present-day values have been assumed.
3. Changes in R could be expected because aircraft populations may change, and improved turnoffs may be added to existing runways. In presenting the 1970 predictions in the Airport Capacity Handbook, better-than-average turnoffs were assumed, and values of R were calculated accordingly by use of the technique already described.
4. Changes in T (departure/departure spacing) can be expected because of improved navigational facilities, improved ground radar, and possibly some automatic sequencing facilities. However, the changes will not, in all probability, be very dramatic since individual aircraft performance (relative aircraft speeds) will still be a basic limitation. Table 6-I lists the spacings between successive departures by class estimated for 1970-75.
5. Changes in F (departure/arrival spacing) can also be expected to occur for the same reasons as given for T. Also some added flexibility should be expected if the present 2 or 3 mile rule for departure release gives way to a rule based on aircraft-class sequencing. For example, in the sequence of a Class D or E aircraft on departure followed by the same class of aircraft on arrival, the present 2-mile release rule is rather restrictive. With good radar monitoring and some automation of the sequencing process, this rule could well

be reduced for these types of aircraft in sequence. Table 6-II lists estimates for F by class sequences for 1970-75.

FIM

Arrival capacity. A, arrival followed by arrival. A great potential exists for improving arrival intervals by 1970. The present fixed rule of 3-mile minimum spacings could well be replaced by a more flexible system of time spacing dependent upon aircraft class sequencing similar to that described for F above. Also, some automatic aids for the controller could well assist him in carrying out the physical operations of sequencing such that improved accuracy would result.

Table 6-III presents estimates of arrival spacings for 1970-75. The figures in parenthesis are the equivalent mileage spacing distances, and where intervals are for fast followed by slower aircraft the common path length in miles at which the closest spacing exists is also given.

4. TAXIING AIRCRAFT CROSSING RUNWAYS

From work on other projects where actual airport configurations were being analyzed, it became evident that the Airport Capacity Handbook (reference 11) should provide a means of evaluating the situations resulting when taxiing aircraft must cross active runways. At airports with close parallel runways, all traffic using the runway furthest from the terminal must cross the inner runway. Does this affect capacity, and how does it affect delay?

Appendix F indicates that the SAM model can be used to analyze the runway crossing situation. Accordingly numerous analyses were made with the SAM model, to devise a

handbook type analysis, now included in reference 11, and from which one can determine:

a. The number of aircraft turning off a runway at various exits (to permit calculating the rate of movement on a taxiway leading to an active runway).

b. The maximum rate of crossings permitted without disrupting landings and takeoffs on the runway to be crossed.

c. The location of a crossing point to permit obtaining a specific crossing rate without disrupting runway operations.

d. The average delay resulting to aircraft crossing an active runway.

5. APPLICATIONS OF CAPACITY AND DELAY ANALYSIS

The technique has been expanded through actual application by AIL on other contracts to airports such as O'Hare International Airport in Chicago, reference 12, and St. Louis Lambert Airport, reference 13. The knowledge gained in these actual applications combined with the work of this contract has resulted in the following improvements in the technique:

1. A step-by-step pattern has been developed for an economic analysis. It covers all the aspects to be examined such as airspace, ground traffic flow, runway use, weather effect, demand forecasts, facility costs, and operating costs. This is detailed in the Airport Capacity Handbook.

2. The method of combining hourly movement rates has been determined, in order to relate them properly to the SAM or FIM analyses. Appendix B discusses this.

3. In an airport analysis, particularly for future years it is usual that the least efficient runway configurations will become heavily overloaded. With the SAM or FIM models, delay becomes infinite for overload conditions. Thus a time dependent analysis is needed in such a case to accurately

determine delay. In Appendix A, a time dependent model is presented which can be used to study such overload conditions. From a practical standpoint, an engineer analyzing an airport has neither the ready understanding nor access to computer facilities to use such a model. Consequently, an empirical evaluation has been developed with the aid of the model of Appendix A to analyze severe overload conditions. The technique included in the Airport Capacity Handbook, can readily be applied, and has been found to provide a reasonable and conservative approximation of delay during overload conditions. The formulation follows:

VFR Procedure for Summing Excess Delay

Let λ_{TC} = departure capacity at 5-minute delay

λ_{Tn} = departure demand in hour n

Problem: Find delay in minutes

(1) First Hour

$$\text{Delay} = \lambda_{T1} \times 5$$

Because of the slow delay build-up, the first hour is only a 5-minute average delay.

(2) Second Hour

$$\text{Delay} = \lambda_{T2} \times 5 + \left[\left(\frac{60}{\lambda_{TC}} - \frac{60}{\lambda_{T2}}\right) \lambda_{T2}\right] \frac{\lambda_{T2}}{2}$$

$$\text{Let } m_n = \left(\frac{60}{\lambda_{TC}} - \frac{60}{\lambda_{Tn}}\right) \lambda_{Tn}$$

m_x approximates the delay build-up during hour x

(3) Third Hour

$$\text{Delay} = \lambda_{T3} \times 5 + \left(m_2 + m_3\right) \frac{\lambda_{T3}}{2}$$

(4) n^{th} Hour

$$\text{Delay} = \lambda_{Tn} \times 5 + \left(m_2 + \ldots m_n\right) \frac{\lambda_{Tn}}{2}$$

(5) Last Hour

$$\lambda_T \text{ demand} < \lambda_{TC}; \text{ delay} = \frac{m_2 + m_3 \ldots m_n}{2} \lambda_T \text{ demand}$$

IFR Procedure for Summing Excess Delay

The analysis of IFR operations is similar to that of VFR except that the initial arrival delay is included.

Let λ_{TC} = departure capacity at 5-minute delay

λ_{Tn} = departure demand in hour n

λ_{Sn} = total demand in hour n

Problem: Find delay in minutes

(1) First Hour

$$\text{Delay} = \lambda_{S1} \times 5$$

(2) Second Hour

$$\text{Delay} = \lambda_{S2} \times 5 + \left[\left(\frac{60}{\lambda_{TC}} - \frac{60}{\lambda_{T2}}\right) \lambda_{T2}\right] \frac{\lambda_{T2}}{2}$$

$$\text{Let } m_n = \left[\left(\frac{60}{\lambda_n} - \frac{60}{\lambda_{Tn}}\right) \lambda_{Tn}\right]$$

(3) Third Hour

$$\text{Delay} = \lambda_{S3} \times 5 + (m_2 + m_3) \frac{\lambda_{T3}}{2}$$

(4) n^{th} Hour

$$\text{Delay} = \lambda_{Tn} \times 5 + (m_2 + \ldots m_n) \frac{\lambda_{Tn}}{2}$$

(5) Last Hour

$$\lambda_T \text{ demand} < \lambda_{TC}; \text{ delay} = \frac{m_2 + \ldots m_n}{2} \lambda_T \text{ demand}$$

6. ECONOMIC ANALYSIS

The final chapter of the Airport Capacity Handbook (reference 11) details the procedure to be used for an economic analysis of an airport design. This is basically similar to the technique developed in the previous work.

TABLE 6-I

T, AVERAGE MINIMUM INTERVAL BETWEEN SUCCESSIVE DEPARTURES ON SAME RUNWAY (IFR ESTIMATE FOR 1970)

Aircraft Class (Departure)	Aircraft Class (Departure)	T* (Sec)
A	A	68
A	B	70
A	C	55
A	D + E	64
B	A	86
B	B	60
B	C	50
B	D + E	45
C	A	91
C	B	67
C	C	52
C	D + E	48
D + E	A	96
D + E	B	90
D + E	C	68
D + E	D + E	52

*T = average minimum interval between successive departures.

TABLE 6-II

F, AVERAGE MINIMUM INTERVAL REQUIRED FOR DEPARTURE RELEASE IN FRONT OF AN INCOMING ARRIVAL (IFR ESTIMATE FOR 1970)

Aircraft Class (Departure)	Aircraft Class (Arrival)	F* (Sec)
A	A	55
A	B	64
A	C	67
A	D	69
A	E	73
B	A	51
B	B	41
B	C	43
B	D	45
B	E	54
C	A	55
C	B	44
C	C	42
C	D	39
C	E	48
D	A	56
D	B	50
D	C	38
D	D	35
D	E	39
E	A	61
E	B	50
E	C	48
E	D	35
E	E	39

*F = average minimum interval required for departure release.

TABLE 6-III

A, AVERAGE MINIMUM INTERVAL BETWEEN SUCCESSIVE ARRIVALS ON SAME RUNWAY (IFR ESTIMATE FOR 1970)

Aircraft Class (Arrival)	Aircraft Class (Arrival)	A* (sec)	Closest Distance (n mi)	Common Path Length (n mi)
A	A	83	3.0	-
A	B	86	2.5	8.0
A	C	95	2.5	7.0
A	D + E	103	2.0	6.0
B	A	83	3.0	-
B	B	76	2.5	-
B	C	79	2.3	7.0
B	D + E	91	1.9	6.0
C	A	83	3.0	-
C	B	76	2.5	-
C	C	82	2.3	-
C	D + E	88	1.9	6.0
D + E	A	83	3.0	-
D + E	B	76	2.5	-
D + E	C	82	2.3	-
D + E	D + E	83	1.9	-

*A = average minimum interval between successive arrivals.

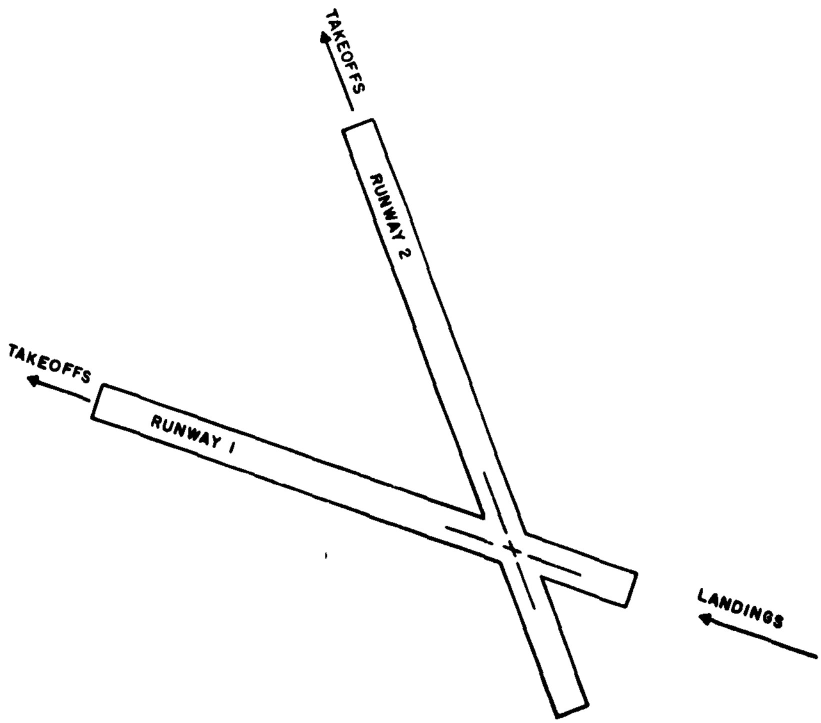


FIGURE 6-1. INTERSECTING RUNWAY WITH CLOSE INTERSECTIONS

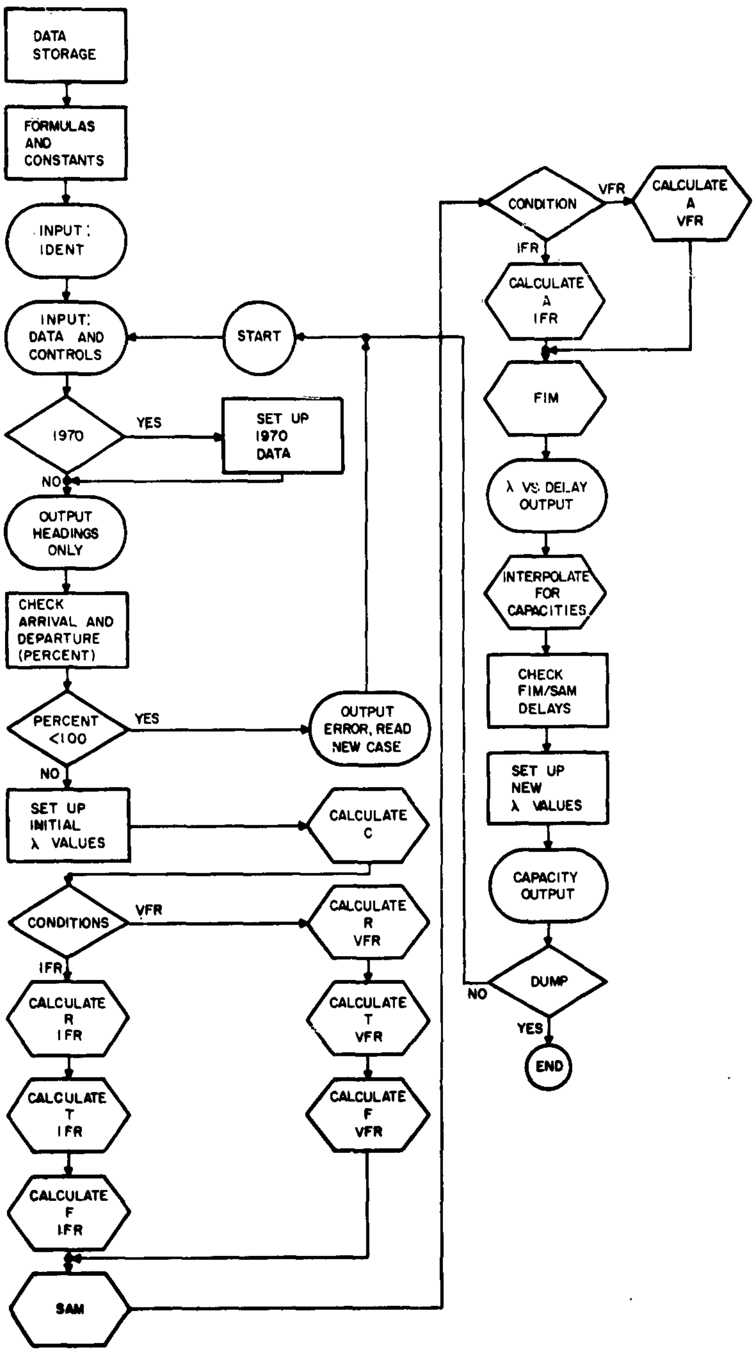


FIGURE 6-2. SIMPLIFIED FLOW DIAGRAM OF AIRPORT CAPACITY PROGRAM (IBM 7090 COMPUTER)

RUN NO. 9067. 1. 1. 1. | VFR X | RATIO 1.00 | RUNWAY RATING 60.0000 60.0000

LAN R	TAK R	TMR	SAM2X	LAN R1	LANR2	TAK R1	TAK R2	FIM LM
10.0	10.0	20.0	36.6	10.0	0.	5.0	5.0	16.0
13.0	13.0	26.0	52.5	13.0	0.	6.5	6.5	17.9
16.0	16.0	32.0	75.1	16.0	0.	8.0	8.0	19.8
18.0	18.0	36.0	96.9	18.0	0.	9.0	9.0	21.2
20.0	20.0	40.0	128.9	20.0	0.	10.0	10.0	22.6
21.0	21.0	42.0	151.2	21.0	0.	10.5	10.5	23.3
22.0	22.0	44.0	180.3	22.0	0.	11.0	11.0	24.0
23.0	23.0	46.0	220.0	23.0	0.	11.5	11.5	24.8*
24.0	24.0	48.0	277.2	24.0	0.	12.0	12.0	26.1*
25.0	25.0	50.0	367.2	25.0	0.	12.5	12.5	27.8*
26.0	26.0	52.0	530.0	26.0	0.	13.0	13.0	29.5*
27.0	27.0	54.0	914.5	27.0	0.	13.5	13.5	31.4*
28.0	28.0	56.0	2950.2	28.0	0.	14.0	14.0	33.4*
29.0	29.0	58.0	7777.0	29.0	0.	14.5	14.5	35.5*
30.0	30.0	60.0	3158.1	30.0	0.	15.0	15.0	37.8*
31.0	31.0	62.0	3257.1	31.0	0.	15.5	15.5	40.2*
32.0	32.0	64.0	3357.7	32.0	0.	16.0	16.0	42.9*
33.0	33.0	66.0	3458.4	33.0	0.	16.5	16.5	45.8*
34.0	34.0	68.0	3559.1	34.0	0.	17.0	17.0	48.9*
35.0	35.0	70.0	3600.0	35.0	0.	17.5	17.5	52.4*
38.0	38.0	76.0	3600.0	38.0	0.	19.0	19.0	65.0*
40.0	40.0	80.0	3600.0	40.0	0.	20.0	20.0	75.9*
42.0	42.0	84.0	3600.0	42.0	0.	21.0	21.0	89.9*
44.0	44.0	88.0	3600.0	44.0	0.	22.0	22.0	108.4*
46.0	46.0	92.0	3600.0	46.0	0.	23.0	23.0	134.0*
47.0	47.0	94.0	3600.0	47.0	0.	23.5	23.5	150.8*
48.0	48.0	96.0	3600.0	48.0	0.	24.0	24.0	171.4*
49.0	49.0	98.0	3600.0	49.0	0.	24.5	24.5	197.2*
50.0	50.0	100.0	3600.0	50.0	0.	25.0	25.0	230.5*
51.0	51.0	102.0	3600.0	51.0	0.	25.5	25.5	275.2*
52.0	52.0	104.0	3600.0	52.0	0.	26.0	26.0	338.3*
53.0	53.0	106.0	3600.0	53.0	0.	26.5	26.5	434.0*
54.0	54.0	108.0	3600.0	54.0	0.	27.0	27.0	596.6*
55.0	55.0	110.0	3600.0	55.0	0.	27.5	27.5	933.4*
56.0	56.0	112.0	3600.0	56.0	0.	28.0	28.0	2048.7*
57.0	57.0	114.0	3600.0	57.0	0.	28.5	28.5	77777.0*

CAP.SAM2X= 38.89

CAP.FLM= 44.91

KEY:

- VFR X: INTERSECTING RUNWAYS VFR CONDITIONS
- RATIO: $\frac{\text{ARRIVAL RATE}}{\text{DEPARTURE RATE}}$
- RUNWAY RATING: RATING IN ORDER OF RUNWAYS (1, 2, etc.)
- LAN R: AIRPORT ARRIVAL RATE
- TAK R: AIRPORT DEPARTURE RATE
- TMR: TOTAL MOVEMENT RATE (LAN R + TAK R)
- SAM 2X: AVERAGE DEPARTURE DELAY IN SECONDS UP TO 7777.0 (INFINITE DELAY). THEN HOURLY UTILIZATION IN SECONDS UP TO 3600 (100%).
- LAN R1: ARRIVAL RATE, RUNWAY 1
- LAN R2: ARRIVAL RATE RUNWAY 2
- TAK R1: DEPARTURE RATE RUNWAY 1
- TAK R2: DEPARTURE RATE RUNWAY 2
- FIM LM: AVERAGE ARRIVAL DELAY IN SECONDS UP TO 77777.0 (INFINITE DELAY)
- CAP. SAM 2X: TMR CAPACITY BASED ON DELAY FOR POPULATION (2 MINS HERE, A + B = 0)
- CAP. FLM: LAN R CAPACITY BASED ON DELAY FOR POPULATION.

FIGURE 6-3. EXAMPLE OF COMPUTER OUTPUT

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12. Analysis of Capacity and Staging of Runway Construction at Chicago O'Hare Airport, AIL Report No. 8914-1 dated April 1962, and Supplement AIL Report No. 2742-1 dated January 1963.

13. M. A. Warskow, et al., "Airport Facilities for General Aviation," Report No. 1400-1, Airborne Instruments Laboratory, Report No. 1400-1, November 1952.

VIII. CONCLUSIONS

The SAM and FIM mathematical models, together with the service times observed or interpreted from the field data, reproduce airport operations in terms of movement rate versus delay for the following situations:

1. Single, parallel, intersecting, and open V runway configurations,
2. VFR and/or IFR operations,
3. Runway/taxiway intersection crossings.

The model analysis is based on observed safe operating practices. Validation testing has shown that the combined model/input-generation program, measures delay with sufficient accuracy, and represents safe operations with sufficient faithfulness to constitute a standard of evaluation for operational performance and airport design.

It is noted that assuming a Poisson arrival input (the bases for the models) is a desirable public policy (properly conservative).

Field data has indicated that, for similar runway configurations, handling similar types of aircraft in VFR, there is little or no difference between the behavior of experienced controllers and pilots from one airport to another, other than very short-term effects. Presumably this holds in IFR, but here the evidence is not yet as clear. Because of this conformity, the technique provides a common national standard for evaluating airport design.

With regard to capacity, airspace restrictions and limitations have more effect on departures than on arrivals. The effect on arrivals tends to be more in terms of economics

than capacity. Lack of airspace routings on departures requires longer spacing intervals between successive departures. This, in turn, causes greater departure delays, which tend to clog the airport. For arrivals, similar airspace limitations will not necessarily reduce the arrival rate, but may cause devious routings, off-optimum speeds, etc., which have economic repercussions.

Airport altitude has little effect on airport capacity except for increased runway occupancy for landing aircraft. This effect can be calculated.

Runway occupancy time can now be calculated to a reasonable degree of accuracy for any given runway/turnoff configuration. The technique developed should be regarded as interim since it requires further refinement to provide information on the use of turnoffs. Use of the technique has provided some insight into the optimum placing of runway turnoffs, and it is considered that some extra work on this aspect would provide valuable data. The model/input-generation program can be used to evaluate the effect of altering runway occupancy times.

The techniques developed and the data accumulated should have some bearing on the airspace and airport simulation experiments presently being conducted by the FAA. It has been determined that the steady-state models are valid in analyzing traffic flows, which vary from hour to hour provided that certain stated rules of application are applied. It therefore follows that the techniques would provide a good starting point on which to base airspace/airport simulation studies, and the spacing data can be used directly in the actual simulation.

In preparing forecasts for economic analyses of airports it is necessary to project known figures of airport movements. At some airports this is very difficult because present statistics available from the FAA Air Traffic Activity Reports are sometimes inadequate since helicopter and training flying (touch-and-go movements) are not separated. Helicopters very rarely use existing fixed-wing runways, and though their operations place a load on controllers they do not impose a runway capacity problem. At some airports touch-and-go operations (both helicopters and fixed-wing aircraft) are often performed away from the primary runways and are counted as two movements (an arrival and a departure). Again, this may be valid in determining controller work load but in terms of runway usage it greatly distorts the airport movements.

Aircraft population greatly affects airport capacity. Present tower statistics are inadequate to accurately gage whether any one airport is working at or close to capacity.

Since completing the handbook it has become apparent that, spacing intervals of jet aircraft have been somewhat reduced in both VFR and IFR. Also in some instances there has been local relaxation of rules governing the operation of intersecting runways.

IX. RECOMMENDATIONS

The models and parameters input techniques should be adopted as a standard for evaluating airport design and as a baseline for future simulation studies. It is recommended for use in evaluating the traffic processing capability of an operating facility.

There should be periodic reviews and data taking relevant to the models to periodically amend the handbook as new aircraft and control techniques are introduced. This will also update the data library available for simulation experiments.

A review period of two to three years is suggested to cover 2 airports for VFR operations, and 2 airports for IFR operations. These airports should be selected on the basis of high movement rates and other interesting operating situations which have developed since the preceding period. Since data taking on this project ended in early 1962, the first review should occur during 1964-1965.

Future studies should examine the use of the techniques for applying flow control and further work should be performed on the economic aspects of airport design and traffic control techniques.

The question of runway occupancy and placement of turnoffs should be further investigated:

1. To compile a library of data on aircraft performance on the runway subject to parameters such as rain, runway length, position, and type of turnoffs.

2. To provide a more accurate and graded (but essentially simple) technique for estimating runway time and use of exits.

3. To provide a scientific basis for determining optimum turnoff locations, either right-angled exits or high-speed turnoffs.

Aircraft movement statistics taken from control towers should be modified to separate helicopter and touch-and-go training operations. This should be done as soon as possible.

Steps should be initiated to improve the quality of airport movement statistics by listing operations by time, class of aircraft, and runway used. It is recognized that this is more complex than present requirements, and requires that an improved data recording technique (possibly semiautomatic) be made available to tower personnel.

APPENDIX A

TIME-DEPENDENT NONSTATIONARY RUNWAY MODEL

1. ARBITRARY ARRIVAL AND SERVICE-TIME DISTRIBUTIONS

The purpose of this appendix is to show the development of a time-dependent scheme for a single-server first-come-first-served queuing model with arbitrary arrival and service-time distributions. The output of this model will be the distribution and moment of delay.

The model. The time axis ($t > 0$) is divided into discrete quantities of equal size Δt. A single server initiates and terminates service of any customer only at times $t_i = i\Delta t$ and $t_T = T\Delta t$, where $T \geq i \geq 0$. Customers may arrive in the time interval t, $i\Delta t < t \leq (i + 1)\Delta t$. If the system is empty and the next customer arrives at time $t = k\Delta t$, his service will initiate at time $t_i = k\Delta t$ and this customer will be undelayed. Should the customer arrive within the time t, $(k - 1)\Delta t < t < k\Delta t$, then he will incur some delay.

We are concerned, basically, with the delay that a customer might incur. In particular, let us think of a film that has been exposed at the time $i\Delta t$, $i = 1, 2, 3\ldots$ If the viewer thinks of himself as the next customer, then what he sees will be his delay as a function of time at the time that he is viewing.

Equations of delay. Essentially, the delay to be incurred at time $(i + 1)\Delta t$ will be the delay that was incurred at time $i\Delta t$ minus the interval of time Δt, plus the total service of all customers that have arrived in the same interval. Letting $w(n, i)$ be the probability of delay $n\Delta t$ at time $i\Delta t$, $a(n)$, the probability of n arrivals in time Δt, and $s(n)$ the

probability that service of a customer is $n\Delta t$, the equations of delay are

$$w(0, i+1) = a(0)\left[w(1, i) + w(0, i)\right]$$

$$w(1, i+1) = a(0)\, w(2, i) + a(1)\, s(1)\left[w(1, i) + w(0, i)\right]$$

$$w(2, i+1) = a(0)\, w(3, i) + a(1)\left\{s(1)\, w(2, i) + s(2)\left[w(1, i) + w(0, i)\right]\right\} + a(2)\, s^{x}(2)\left[w(1, i) + w(0, i)\right]$$

where $s^{x}(n)$ is a convolution. Generalizing, let $s^{i}(n)$ be the probability that i customers have a total service of $n\Delta t$, then (noting that $s^{i}(n) = 0$ for $i > n$)

$$\begin{aligned} w(n, i+1) = {} & a(0)\, w(n+1, i) \\ & + a(1)\left\{\sum_{K=1}^{n-1} s^{1}(K)\, w(n+1-K, i) + s^{1}(n)\left[w(1, i) + w(0, i)\right]\right\} \\ & + a(2)\left\{\sum_{K=1}^{n-1} s^{2}(K)\, w(n+1-K, i) + s^{2}(n)\left[w(1, i) + w(0, i)\right]\right\} \\ & + \ldots \\ & + a(n)\, s^{n}(n)\left[w(1, i) + w(0, i)\right] \end{aligned}$$

Let

$$b(n) = \sum_{j=1}^{\infty} a(j)\, s^{s}(n)$$

$$b(0) = a(0)$$

Then

$$w(0, i+1) = b(0)\left[w(1, i) + w(0, i)\right]$$

$$w(n, i+1) = b(0)\, w(n+1, i) + \sum_{K=1}^{n-1} b(K)\, w(n+1-K, i) + b(n)\left[w(1, i) + w(0, i)\right] \quad n > 0 \qquad \text{(A-1)}$$

If we let

$$w_i(t) = \sum_{n=0}^{\infty} w(n, i)\, t^n$$

$$b(t) = \sum_{n=0}^{\infty} b(n) t^n$$

the generating function form of equation A-1 is

$$w_{i+1}(t) = \frac{b(t)\left[w_i(t) - w_i(0)\right]}{t} + w_i(0)\, b(0) \qquad \text{(A-2)}$$

For the corresponding cumulative form, let

$$G(n, i) = \sum_{K=n+1}^{\infty} w(K, i)$$

$$B(n) = \sum_{K=n+1}^{\infty} b(K)$$

$$G_i(t) = \sum_{n=0}^{\infty} G(n, i) t^n$$

$$B(t) = \sum_{n=0}^{\infty} B(n) t^n$$

Then from equation A-2

$$G_{i+1}(t) = B(t) + \frac{b(t)\left[G_i(t) - G_i(0)\right]}{t} \qquad \text{(A-3)}$$

or

$$G(n, i + 1) = B(n) + \sum_{K=0}^{n} b(K)\, G(n + 1 - K, i) \tag{A-4}$$

Expression A-4 is very useful for computing purposes since the cumulative form easily lends itself to a truncation criterion.

Moments of delay. By repeated differentiations of equation A-2, or summing in equation A-1, the first three moments of delay are

$$W_1(i + 1) = B_1 + W_1(i) - G(0, i)$$

$$W_2(i + 1) = B_2 + W_2(i) + 2W_1(i) \left[B_1 - 1\right] + G(0, i) \left[1 - 2B_1\right]$$

$$W_3(i + 1) = B_3 + W_3(i) + 3W_2(i) \left[B_1 - 1\right] + 3W_1(i)\left[B_2 - 2B_1 + 1\right] - G(0, i) \left[3B_2 - 3B_1 + 1\right]$$

where

$$W_n(i) = \sum_{K=0}^{\infty} K^n w(K, i); \; B_n = \sum_{K=0}^{\infty} K^n b(n)$$

Minimum interval size. The function $G(n, i) = G(n\Delta t, i\Delta t)$ is computed from equation A-4. Experience has indicated that as $\Delta t \to 0$, the function G approaches a limit for each of its arguments. If $x = \lim n\Delta t, \Delta t, \Delta t \to 0$; $t = \lim i\Delta t, \Delta t \to 0$, $G(x,t)$ represents the continuous solution for $x, t > 0$. The question of interval size is important in order to obtain accurate solution at minimal cost, and so, $| G(x, t) - G(n\Delta t, i\Delta t|$ is small for $n\Delta t = x$, $i\Delta t = t$. The recursive scheme equation A-4 has a very nice criterion.

Specifically, let $b(t)$ be associated with Δt, and $\bar{b}(t)$ be associated with $\frac{\Delta t}{m}$. If $b(t^m) = \left[\bar{b}(t)\right]^m$ for $m > 1$, then any subdivision of Δt will not yield greater accuracy in the function $G(x, t)$ at the corresponding points. Or if $b(t)$ and $\bar{b}(t)$ are associated with Δt, and $\Delta t/m$ such that $b(t^m) = \left[\bar{b}(t)\right]^m$; then $G(n\Delta t, i\Delta t) = \bar{G}(mn \frac{\Delta t}{m}, mi \frac{\Delta t}{m})$ given the same initial condition. The basic computing difference between G and $\bar{G}$ is that the latter requires m time steps of computing, and the former only one. We shall consider the case of $m = 2$.

Case 1:

Let

$$G_o(t) = 0 = \bar{G}_o(t)$$

From equation A-4

$$G_1(t) = \frac{1 - b(t)}{1 - t}$$

whereas

$$\bar{G}_1(t) = \frac{1 - \bar{b}(t)}{1 - t}$$

$$\bar{G}_2(t) = \frac{t\left[1 - \bar{b}(0)\,\bar{b}(t)\right] + \bar{b}(t)\left[\bar{b}(0) - \bar{b}(t)\right]}{t(1 - t)}$$

Forming $\bar{G}_2(t) - G_1(t^2)$, the difference is an odd function if $b(t^2) = \bar{b}^2(t)$.

Case 2:

Let

$$G_o(t) = t^n,\ \bar{G}_o(t) = t^{2n}$$

$$G_1(t) = \frac{1 - t^{n-1}\,b(t)}{1 - t}$$

$$\bar{G}_1(t) = \frac{1 - t^{2n-1}\,\bar{b}(t)}{1 - t}$$

$$\overline{G}_2(t) = \frac{1 - t^{2n-2}\,\overline{b}(t)}{1 - t}$$

Again, $\overline{G}_2(t) - G_1(t^2)$ is an odd function if $b(t^2) = \overline{b}^2(t)$. All initial conditions are linear combinations of cases 1 and 2. In both cases, we have constructed an odd function relating the function G at two different mesh spacings, one being half of the other, which is desired. This result leads to a criterion for minimum interval size.

Moments of b(n). The moments of b can be expressed in terms of the moments of a(n) and s(n). That is

$$B_1 = S_1 A_1$$

$$B_2 = S_1^{\,2}\,(A_2 - A_1) + S_2 A_1$$

$$B_3 = S_1^{\,3}(A_3 - 3A_2 + 2A_1) + 3(A_2 - A_1)\,S_1 S_2 + A_1 S_3$$

Steady-state moments of delay. If the steady-state monents are finite (necessary that $B_1 < 1$), then

$$\lim_{i\to\infty} G(0,\ i) = G(0) = B_1 = S_1 A_1$$

$$\lim_{i\to\infty} W_1(i) = W_1 = \frac{B_2 + G(0)\left[1 - 2B_1\right]}{2(1 - B_1)}$$

$$= \frac{S_1^{\,2}(A_2 - A_1) + S_2 A_1 + S_1 A_1\,(1 - 2S_1 A_1)}{2(1 - S_1 A_1)}$$

Specifically, let $b(t)$ be associated with Δt, and $\overline{b}(t)$ be associated with $\frac{\Delta t}{m}$. If $b(t^m) = \left[\overline{b}(t)\right]^m$ for $m > 1$, then any subdivision of Δt will not yield greater accuracy in the function $G(x, t)$ at the corresponding points. Or if $b(t)$ and $\overline{b}(t)$ are associated with Δt, and $\Delta t/m$ such that $b(t^m) = \left[\overline{b}(t)\right]^m$; then $G(n\Delta t, i\Delta t) = \overline{G}(mn \frac{\Delta t}{m}, mi \frac{\Delta t}{m})$ given the same initial condition. The basic computing difference between G and $\overline{G}$ is that the latter requires m time steps of computing, and the former only one. We shall consider the case of $m = 2$.

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From equation A-4

$$G_1(t) = \frac{1 - b(t)}{1 - t}$$

whereas

$$\overline{G}_1(t) = \frac{1 - \overline{b}(t)}{1 - t}$$

$$\overline{G}_2(t) = \frac{t\left[1 - \overline{b}(0)\, \overline{b}(t)\right] + \overline{b}(t)\left[\overline{b}(0) - \overline{b}(t)\right]}{t(1 - t)}$$

Forming $\overline{G}_2(t) - G_1(t^2)$, the difference is an odd function if $b(t^2) = \overline{b}^2(t)$.

Case 2:

Let

$$G_o(t) = t^n, \quad \overline{G}_o(t) = t^{2n}$$

$$G_1(t) = \frac{1 - t^{n-1}\, b(t)}{1 - t}$$

$$\overline{G}_1(t) = \frac{1 - t^{2n-1}\, \overline{b}(t)}{1 - t}$$

$$\overline{G}_2(t) = \frac{1 - t^{2n-2}\,\overline{b}(t)}{1 - t}$$

Again, $\overline{G}_2(t) - G_1(t^2)$ is an odd function if $b(t^2) = \overline{b}^2(t)$. All initial conditions are linear combinations of cases 1 and 2. In both cases, we have constructed an odd function relating the function G at two different mesh spacings, one being half of the other, which is desired. This result leads to a criterion for minimum interval size.

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Steady-state moments of delay. If the steady-state monents are finite (neceε ary that $B_1 < 1$), then

$$\lim_{i \to \infty} G(0,\ i) = G(0) = B_1 = S_1 A_1$$

$$\lim_{i \to \infty} W_1(i) = W_1 = \frac{B_2 + G(0)\left[1 - 2B_1\right]}{2(1 - B_1)}$$

$$= \frac{S_1^{\,2}(A_2 - A_1) + S_2 A_1 + S_1 A_1\,(1 - 2S_1 A_1)}{2(1 - S_1 A_1)}$$

$$\lim_{i\to\infty} W_2(i) = W_2 = \frac{3W_1\left[B_2 - 2B_1 + 1\right] + B_3 + G(0)\left[3B_1 - 1 - 3B_2\right]}{3(1 - B_1)}$$

Note that the moments are really a summation that approximates, for the continuous case, an integral. Despite the fact that $G(n, i)$ may be exact, i.e., Δt satisfies a minimum criterion, there is still an error in numerical integration that should be corrected.

Example 1:

Let

$$a(n) = \frac{e - \lambda\Delta t(\lambda\Delta t)^n}{n} \qquad \bar{a}(n) = \frac{e^{-\frac{\lambda\Delta t}{-2}}}{n!}\left(\frac{\lambda\Delta}{2}\right)^n$$

$$s(1\Delta t) = 1 \qquad \bar{s}(2\Delta t/2) = 1$$

$$b(t) = e^{-\lambda\Delta t(1-t)} \qquad \bar{b}(t) = e^{\frac{\lambda\Delta t}{-2}(1 - t^2)}$$

In this case $b(t^2) = \bar{b}^2(t)$

Example 2:

For the Poisson distribution,

$$A_1 = \lambda\Delta t, \; A_2 = (\lambda\Delta t)^2 + (\lambda\Delta t)$$

$$A_3 = (\lambda\Delta t)^3 + 3(\lambda\Delta t)^2 + \Delta t$$

In the previous considerations, $W_n(i)$ has been expressed in terms of intervals of Δt. If we normalize these moments and the service distribution in terms of unit space, we find, for steady state

$$G(0) = \lambda S_1$$

$$W_1 = \frac{\lambda S_2}{2(1 - \lambda S_1)} + \frac{\lambda S_1 \Delta t}{2}$$

$$W_2 = \frac{\lambda^2 S_2^{\,2}}{2(1 - \lambda S_1)^2} + \frac{\lambda S_3}{3(1 - \lambda S_1)} + \frac{\lambda S_2 \Delta t}{2(1 - \lambda S_1)} + \frac{\lambda S_1(1 + \lambda S_1)(\Delta t)^2}{6(1 - \lambda S_1)}$$

since $\Delta t \to 0$, W_1 and W_2 each approach a limit. However, for any Δt, there is an error term for each of W_1 and W_2.

Consider

$$\overline{W}_n(i) = \sum_{K=1}^{\infty} (K - 1)^n w(n, i)$$

Then

$$\overline{W}_1(i) = W_1(i) = G(0, i)$$

$$\overline{W}_2(i) = W_2(i) - 2W_1(i) + G(0, i)$$

As $i \to \infty$, and for normalized units

$$\overline{W}_1 = W_1 - G(0)\ \Delta t$$

$$\overline{W}_2 = W_2 - 2W_1\ \Delta t + G(0)(\Delta t)^2$$

then

$$\frac{W_1 + W_1}{2} = \frac{\lambda S_2}{2(1 - \lambda S_1)}$$

$$\frac{\overline{W}_2 + W_2}{2} = \frac{\lambda^2 S_2^{\,2}}{2(1 - \lambda S_1)} + \frac{\lambda S_3}{3(1 - \lambda S_1)} + \frac{\lambda S_1(1 + \lambda S_1)}{6} (\Delta t)^2$$

To remove an error term of order Δt in the moments,

$$W_1(i + 1) = W_1(i + 1) - \frac{G(0,\ i + 1)}{2}$$

$$W_2(i + 1) = W_2(i = 1) - W_1(i + 1)$$

These corrections can be applied only after the basic iteration scheme on page A-2 has been applied for all time steps.

2. POISSON ARRIVAL DISTRIBUTIONS

In the first part of this appendix the theory is presented for a time-dependent queuing model with arbitrary arrival and service-time distributions. In this part, the theory is specialized to a Poisson arrival distribution. In addition certain details regarding the numerical computation are given.

Computing procedure for b(n). The b(n) are defined as

$$b(n) = \sum_{j=1}^{\infty} a(j)\ s^j(n) \quad n > o$$

$$b(o) = a(o)) \qquad n = o \qquad \text{(A-5)}$$

where $s^j(n)$ refers to the j^{th} convolution of the service-time distribution s(n), and $s^j(n)$ can be interpreted as the probability that j customers have a total service of $n\Delta t$. Note that $s^1(n) = s(n)$.

The arrival rate in time Δt is λ, and the Poisson distribution can be defined as

$$a(o) = e^{-\lambda}$$
$$a(n) = \frac{\lambda}{n} a(n - 1) \quad n > o \tag{A-6}$$

Given a prescribed truncation criterion ϵ, define N_A as the smallest integer such that

$$\sum_{n=N_A}^{\infty} a(n) < \epsilon$$

Associated with the service-time distribution, there will be two integers N_1 and N_2 such that N_1 is the smallest integer for $s(n) > o$ [that is, $s(n) = o$ for $n < N_1$] and N_2 is the largest integer for $s(n) > o$ [that is, $s(n) = o$ for $n > N_2$]. The convolutions of $s(n)$ can then be computed as

$$s^j(n) = \sum_{j=\max(1, N_1)}^{j=\min(n, N_2)} s^{j-1}(n - j)\, s(j) \quad j \geq 2$$
$$= o \text{ if } \max(1, N_1) > \min(n, N_2) \tag{A-7}$$

Note that $s^i(n) = o$ for $n < iN_1$, $n > iN_2$. In the computing scheme to be described, it is necessary to retain only $s(n)$, $s^{i-1}(n)$, $s^i(n)$; the values of $s^{i+1}(n)$ being written over the values $s^{i-1}(n)$. The following is the scheme used in the computer:

1. Using equation A-6, compute $a(n)$, $o \leq n \leq N_A$

2. Let $b(n) = s(n)\ a(1)\quad N_1 \leq n \leq N_2$

$$= o \qquad n < N_1;\ n > N_2$$

3. $b(o) = a(o)$

   $B(o) = 1 - b(o)$ [$B(n)$ are cumulative distribution]

4. $B(n) = B(n - 1) - b(n) \quad 1 \leq n < N_1$

5. Set $i = 2$ [i referring to the order of the convolution of $s^i(n)$, define $s^1(n) = s(n)$]

6. Using equation A-7, find $s^i(n)$, $iN_1 \leq n \leq iN_2$

7. Let $b(n) = a(i)\ s^i(n) + b(n)$, $iN_1 \leq n \leq N_2$

8. Let $B(n) = B(n - 1) - b(n)$, $(i - 1) \cdot N_1 + 1 \leq n \leq iN_1$

9. If $B(iN_1) < \epsilon$, all of the $b(n)$, $B(n)$ have been defined; further contributors will be small for small ϵ, then define $N_B = iN_1$ and the routine is finished.

10. If $i \geq N_A$

$$B(n) = B(n - 1) - b(n) \quad (i - 1) N_1 + 1 \leq n \leq iN_2$$

   Define N_B as the smallest n such that $B(N_B) < \epsilon$.

11. If N_B cannot be defined by either step 9 or 10, increase i by 1 ($i + 1 \rightarrow i$), and repeat from step 6.

After N_B has been defined, the first and second moments of $b(n)$ are computed. As a check that the distribution of b has been correctly computed, the first moment B_1 can be computed as either

$$B_1 = \lambda S_1 \quad \text{where } S_1 \text{ is the average of service time distributed} \tag{A-8}$$

$$B_1 = \sum_{j=o}^{N_B} BB(j) \tag{A-9}$$

We refer to equation A-8 as the predicted value of B_1 and to equation A-9 as the computed value of B_1.

Computation of delay probability vector. The delay probability vector G(n, i) is computed in the cumulative form and is defined as (iterative scheme):

$$G(n,\ i+1) = B(n) + \sum_{k=o}^{n} b(k)\ G(n+1-k,\ i)$$

(A-10)

Since the smallest non-zero term in B(n) is $B(N_b)$, the upper limit in the summation in equation A-10 is at most N_b. Further, let L_K be defined as the smallest integer such that $G(L_K, i) < \varepsilon$. In the summation in equation A-10, $n + 1 - j \leq L_K$. The delay vector can then be defined as

$$G(n,\ i+1) = B(n) + \sum_{k=\max(o,\ n+1-L_K)}^{\min(n,\ N_B)} b(k)\ G(n+1=k,\ i)$$

(A-11)

for $\max(o,\ n+1-L_K) \leq \min(n,\ N_B)$

$= B(n)$

for $\max(o,\ n+1-L_K) > \min(n,\ N_B)$

The first and second moments of G(n, i + 1), $W_1(i+1)$ and $W_2(i+1)$ are defined

$$W(i+1) = B_1 + W_1(i) - G(o,\ i) \qquad \text{(A-12)}$$

$$W_2(i+1) = B_2 + W_2(i) + 2W_1(i)\left[B_1 - 1\right] + G(o,\ i)\left[1 = 2B_1\right]$$

(A-13

As an alternative to equation A-12, the first moment can be defined as

$$W_1(i + 1) = \sum_{k=o}^{\infty} G(k, i + 1) \qquad \text{(A-14)}$$

Expression A-12 is referred to as average by iteration, and A-13 as average by summation. Occasionally, these are printed on line as a check on the work.

It should be noted that expressions A-12 and A-14 include an error dependent upon the mesh size. After $W_1(i + 1)$ has been computed (for all i), the error can be removed by defining the first moment.

$$W_1(i + 1) - 0.5\ G(o, i + 1)$$

and for the second moment,

$$W_2(i + 1) - W_1(i + 1) + \frac{G(o, i + 1)}{2}$$

Remembering that arrays in the memory are numbered from 1, the computing scheme for equations A-11 through A-13 is:

1. Set n = 1 .
2. Set temp = B(n) -- in the machine the cumulative is BB
3. Set $k_1 = n + 2$
4. Set $k_2 = \min (N_B, k_1)$
5. Set $k_3 = \max (1, k_1 - L_K)$
6. If $k_3 > k_2$ go to step 8, if not, go to step 7
7. For $k_3 \leq j \leq k_2$ i.e., steps 7(a) and 7(b)

(a) $k_4 = k_1 - j$

(b) temp = temp + $b(j)\ u(k_4)$

8. $v(n)$ = temp

if $v(n) \leq \epsilon$ go to step 10, if not go to step 9

9. Repeat steps 2 through 8 for $1 \leq n \leq 5000$

10. If $L_K = n$ find diff $= \max|u(j) = v(j)|\ 1 \leq j \leq n$

11. Set L_K for this vector as n

12. Find from steps 8 and 9, $W_1(i + 1)$ and $W_2(i + 1)$

General discussion. The time axis is divided in increments of equal size. These increments can be grouped together so that for each of these groups the arrival rate λ are constant. When the arrival rate and service-time distribution are known, the distributions $b(n)$ and $B(n)$ can be determined. Within a group of time increments, the delay vector is computed by iteration until either (1) the distribution of delay has been computed for each time increment within a group, or (2) the maximum difference between two successive vectors (with respect to components) is small, in which case the steady-state theory applies. After steady state has been achieved, all successive delay vectors will be identical (with respect to error criteria).

When the delay vectors have been computed for a group of increments, either there are more groups or all computation has been done.

Initially, the system is empty, and so the delay vector is zero.

APPENDIX B

DETERMINATION OF DELAY USING STEADY-STATE MODELS IN NONSTATIONARY SITUATIONS

The mathematical models used to determine airport capacity are steady-state models. They are generally used to determine an operating rate, which provides a specified average delay. This operating rate is called the practical airport capacity.

If the operating rate or demand is known, then the average delay can be determined. However, the demand is a variable that changes from hour to hour. This variation in demand is called the daily distribution of traffic. To accurately determine the total delay experienced throughout a day, it is necessary to make nonstationary or time-dependent analysis. However, the available time-dependent model is both clumsy to use and only rigidly valid for the first-come first-served cases. It was important, therefore, to develop a means of using the steady-state models to determine the delay experienced with a particular daily distribution of traffic demand.

Airport demand is generally forecast first on an annual basis. The forecast can then be broken down to a peak-day and peak-hour forecast. This in turn may be refined into average day peak hour and IFR day peak-hour values, and finally into daily distributions of traffic by hours. In airport design, the steady-state models, are used to determine delay on an hour-by-hour basis for two reasons.

1. Summing the delay. To determine total annual delay (as is done in benefit/cost analyses), it is necessary to analyze the delay on an hour-by-hour basis (rather than on the basis

of one operating rate) for each mode of operation and runway configuration, and then sum the delay by the day and year. How should this hour-by-hour analysis of delay be accomplished to make sure it approximates the actual delay which occurs on a nonstationary or time-dependent basis?

2. Practical capacity compared to peak-hour demand. How can the operating rate at 4-minute delay, which we call the practical airport capacity for large aircraft, be applied to the forecast peak-hour demand?

To determine the answer to these questions, we have investigated an application of a steady-state analysis by comparing the results obtained to those obtained using a time-dependent analysis. The answers that we found approximate the actual situation. Our conclusions are as follows:

1. The summing of delay on a daily and then annual basis will approximate the actual time-dependent occurrence of delay if it is determined by the steady-state method for 2-hour intervals and then summed through the day. The 2-hour intervals are chosen, beginning with the pair of consecutive hours that have the highest average, and then on working pairs of hours on both sides of this peak pair.

2. The proper relationship between the steady-state airport capacity and demand operating rates will vary with the average delay value, which determines the capacity and is as follows:

   (a) For air-carrier operations where Classes A and B aircraft are present, so that 3- or 4-minute average delay determines practical capacity, the average of the two consecutive peak hours of demand should not be greater than the practical capacity operating rate corresponding to the 3- or 4-minute delay value.

   (b) Where only Classes C, D, and E aircraft are in the aircraft population, so that a 2-minute average delay determines practical capacity, the forecast peak-hour

demand should not be greater than the practical operating rate corresponding to the 2-minute delay value.

1. SUMMING THE DELAY

To validate conclusion 1, we have used a time-dependent queuing model with a single-server Poisson arrival distribution and arbitrary serving distribution. An average service time was specified for each hour of the day, and the delay was computed at 30-second intervals. The computed delay was recorded at 5-minute intervals throughout the 24-hour period. Five 24-hour periods with different demand rates and service times were examined. The time-dependent model is described in Appendix A. It is a first-come first-served model and thus is not directly applicable to the mixed runway analyses we are examining here (which are solved by application of SAM). However, it was made comparable by developing service times that would produce a first-come first-served steady-state curve equivalent to the SAM steady-state curve.

The delay was also determined for the same five daily distributions of traffic using the equivalent steady-state models in three different manners. The analysis of the delay is made on four bases:

1. The cumulative delay was determined using the time-dependent model on an hour-by-hour basis.
2. The cumulative delay was determined by taking the steady-state delay for each hourly demand rate and totaling the steady-state delay on this basis.
3. The cumulative delay was determined for a steady-state interval of 2 hours, wherein the demand was averaged for the 2 hours to determine the steady-state delay for that 2-hour period.
4. The cumulative delay on a steady-state basis was determined on a 3-hour interval by taking the demand for 3 hours, averaging it, and finding the steady-state delay for this average demand rate.

On the assumption that the time-dependent models provided the true measure of delay, which was used as a reference and the percentage variation of each of these three steady-state analyses was determined from items 2, 3, and 4. Figure B-1 indicates the results.

The 1-hour interval in Figure B-1 generally gave a delay in excess of the time-dependent delay. The 2-hour interval provided a total that was relatively close, but slightly under the time-dependent delay. The 3-hour interval in all cases gave a relatively low value of total delay.

The five cases shown have a definite trend though each case has a somewhat different traffic distribution. It is concluded that the 2-hour interval most accurately approximates the time-dependent solution and will give reasonable answers if used in any economic analysis. It is important to repeat that the 2-hour interval should be selected on the basis of taking the peak hour and the adjoining highest hour as the first pair and then working in pairs in both directions from these peak hours.

2. PRACTICAL CAPACITY COMPARED TO PEAK-HOUR DEMAND

Because steady-state analysis indicates longer delays at the beginning of an interval than would actually occur, it is important to determine how quickly steady state is achieved, and consequently how closely the steady-state capacity analyses relates to a peak-hour demand. Several runs of the time-dependent model are plotted in Figure B-2. There are four 2-hour intervals shown wherein the steady-state delay during the second hour is near 2 minutes. In each case, the time-dependent delay gradually builds up to the steady-state delay during a 1-hour interval. In other words, since the utilization is relatively low at the delay value chosen, the steady-state condition is achieved during an hour of

operation. Therefore, for airports where a 2-minute delay determines the capacity, the forecast demand peak hour should not be greater than the steady-state forecast operating rate.

Three 3-hour intervals are shown where, at the beginning of the second hour, a 2-hour demand is sufficient to cause a steady-state delay of about 4 minutes. In each case the time-dependent delay achieves steady-state before the end of the first hour. In a typical case (ORD 530/1, the steady-state delay is 4.0 minutes for 0800 - 0900 but the time-dependent delay at the end of that hour has not attained the steady-state delay.

Figure B-2 also shows one case (ORD 430/3) where the steady-state delay starts at 3.4 minutes for 1 hour, goes to 5.1 minutes for the second hour, and 7.5 minutes for the third hour. In this instance the time-dependent delay never does achieve steady state, but always lags the steady-state case and delay, increasing as the steady-state delay increases.

These examples indicate that, at airports where 3 and 4 minutes determine the practical capacity, a 2-hour interval is necessary to achieve steady-state conditions. Therefore, the method of comparing the peak-forecast demand to the airport capacity is to take the peak-hour demand and its adjacent highest hour, average these two hours, and compare them with the steady-state capacity. If the demand is not greater than the airport capacity, the delay will not exceed the specified 3 or 4 minutes.

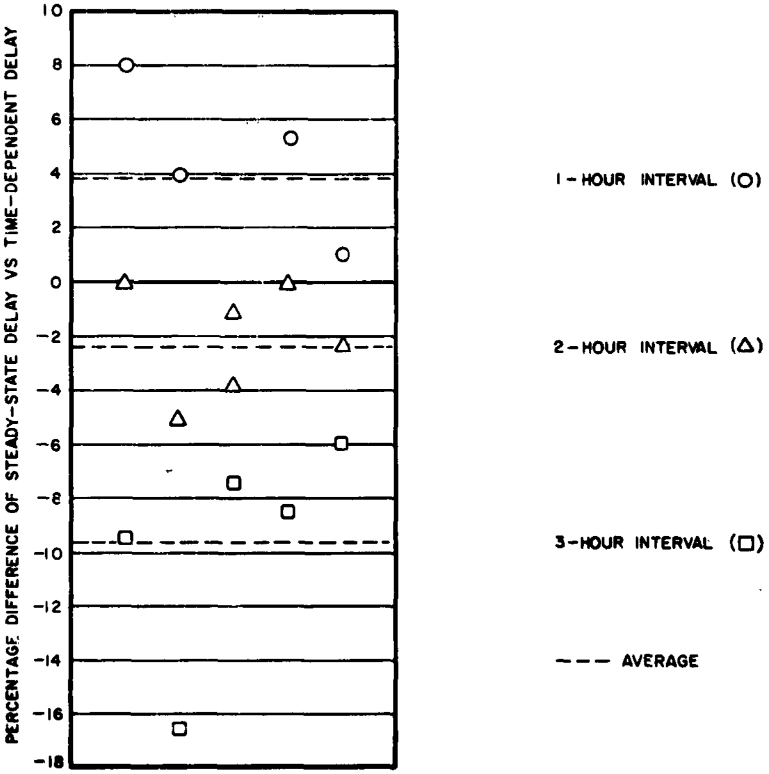


FIGURE B-1. COMPARISON OF STEADY-STATE DELAY WITH TIME-DEPENDENT DELAY

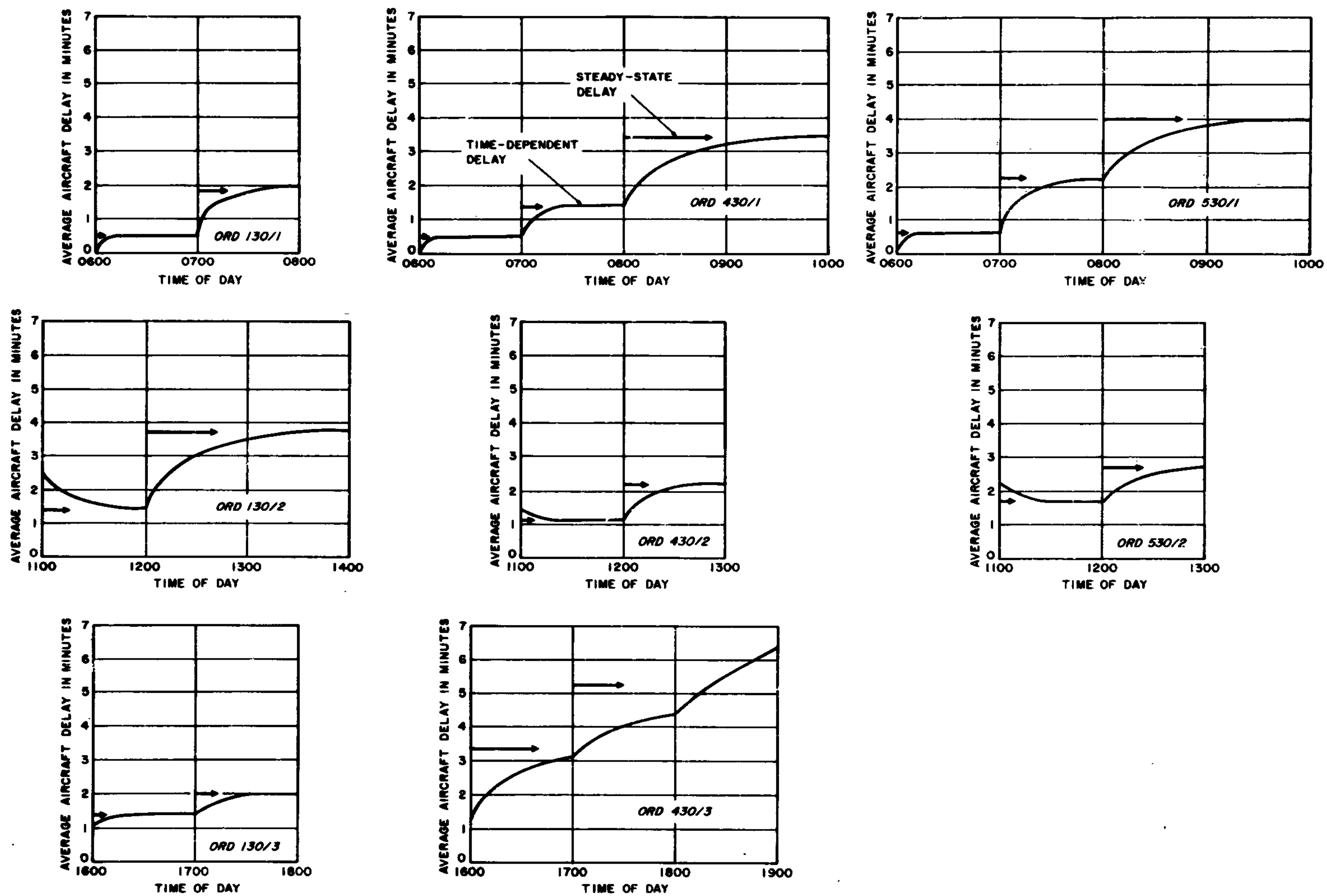


FIGURE B-2. TIME NEEDED TO REACH STEADY-STATE DELAY

APPENDIX C

EFFECTS OF AIRPORT ALTITUDE ON RUNWAY CAPACITY

Because airport altitude is known to affect aircraft landing and takeoff performance, an airport survey was conducted at Stapleton Field, Denver, Colorado (elevation 5331 feet AMSL) to measure aircraft intervals. The intervals of interest were the same as those measured at other airports:

R = Runway occupancy for arrivals

T = Departure-to-Departure interval

A = Arrival-to-Arrival interval

F_{min} = Departure followed by arrival

With the exception of R, the time intervals can only be used for analysis when it is assured that an average minimum exists (Section IV). Because movement rates per runway were rather low at Denver (averaging $\lambda_S = 25$), the number of usable intervals tended to be rather low. However, sufficient data was gathered to conclude the following:

R = Some increase.

T = Little or no change

A = No change

F_{min} = No change

On the basis of aircraft performance it might be expected that the interval T would increase. At higher altitudes engine thrust and propeller efficiency is reduced, thus decreasing acceleration and lengthening the takeoff roll. Some times taken at Denver from start roll to wheels off for various types of aircraft showed that there was an increase in this interval of some 6 seconds for Classes A and B aircraft compared with airports having elevation between 0 and 100 feet AMSL.

However, even though this particular effect was measured, the interval T (which is from start roll of the first departure to start roll of the second departure) was very similar to the data taken at the other airports. Figure C-1 shows the VFR curve of T versus λ_S for pairs of Classes D and E aircraft. This curve is the average value of the data points accumulated at the other airports. The individual points are those obtained at Denver.

Similar data was obtained for the interval A, which is the average minimal interval between successive arrivals at the runway threshold. An example of the Denver data versus other airports is presented in Figure C-2.

For all the intervals of T and A, there was some scatter about the original VFR curves but no significant trend. Therefore, it was concluded that for all practical purposes aiport altitude had no effect on these intervals.

The same was true for the interval F. Here, the minimums measured at Denver for different aircraft class combinations showed no definite trend compared with those taken at other airports. In fact there was a remarkable similarity.

The only definite effect of airport altitude was apparent in the runway occupancy times. This was somewhat complicated by the fact that the runways at Denver generally have rather poor exits. However, despite this, the increased times were noticeable.

Section IV of this report and Appendix C of the Airport Capacity Handbook detail the procedures for determining altitude effects upon runway occupancy based on aircraft performance and the Denver measurements.

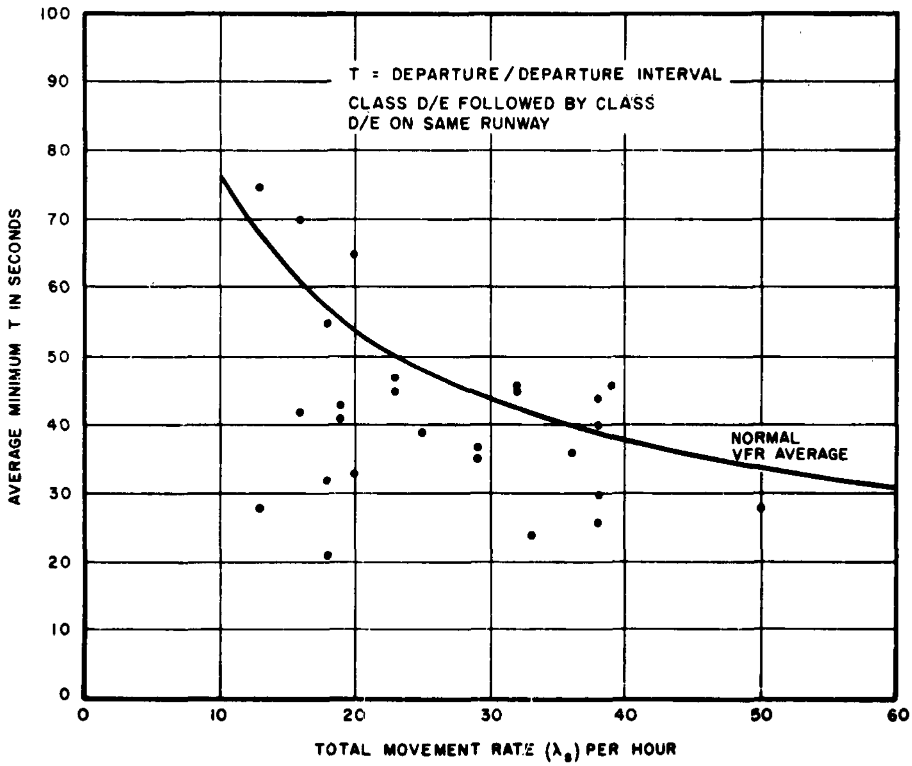


FIGURE C-1. INTERVALS OF T MEASURED AT DENVER

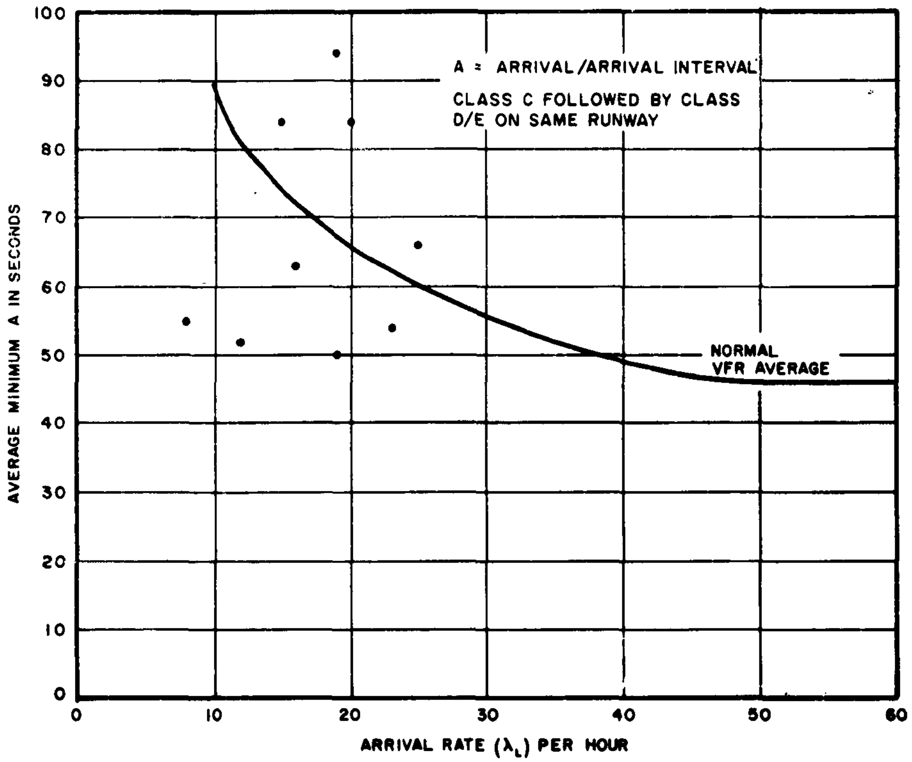


FIGURE C-2. INTERVALS OF A MEASURED AT DENVER

APPENDIX D

ANALYSIS OF AIRCRAFT SPEEDS ON APPROACH

The radar photography of the airport surveillance radar (ASR) scopes taken during the surveys on this project allowed some analysis of aircraft speeds on approach.

For this analysis, the films from the surveys at Washington National, Chicago O'Hare, and Idlewild were used. It was only possible to use data from the days when aircraft were using the ILS for approach and landing, since this required pilots to fly a straight-line track. This requirement meant that, in general, only IFR days could be analyzed, and since in IFR conditions the numbers of Classes D and E aircraft are reduced, most of the speed data could only be obtained for Classes A, B, and C. (No Class A at Washington.)

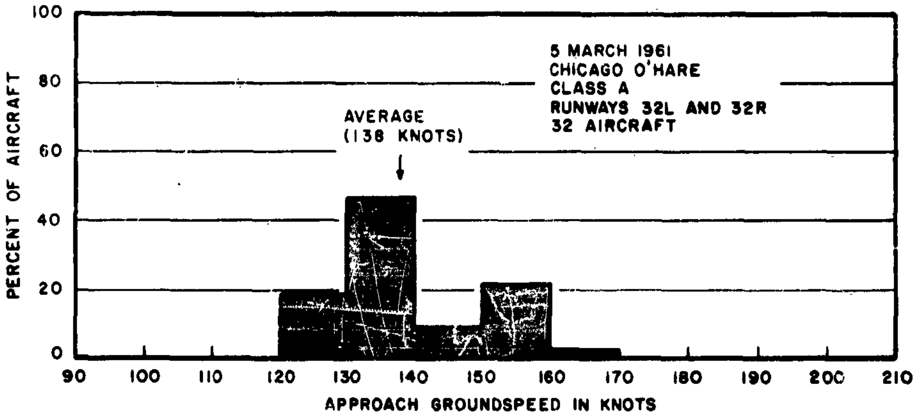
Times were measured between the 10- and 5-mile markers on the radar scope using the clock of the recording system for this purpose.

The final results are shown in Figure D-1. These graphs show percentage of each aircraft class versus approach groundspeed (10-knot increments) for the three airports. Also shown on each graph is the average speed for that class for the particular day at the airport.

It should be emphasized that the speeds shown are groundspeeds, which explains the large variations of the average speeds between different days at the three airports. For example, at Chicago O'Hare on March 6th, the aircraft were approaching the airport on the ILS in the direction of 140 degrees. The surface wind on that day averaged 250 degrees 25 knots gusting to 40 knots. Therefore, there was a strong

tailwind component that boosted the average groundspeeds to 174 knots (Class A) and 161 knots (Class B). The more normal average of groundspeeds for these aircraft in headwinds or crosswinds is 133 to 150 knots (Class A) and 118 and 139 knots (Class B). The situation at Chicago O'Hare on March 6th was somewhat unusual because the aircraft were only using the ILS for approach and were breaking off at the outer marker to circle around to runways 22 and 27 for the actual landing.

Since the aircraft were not landing directly from their ILS approaches, this would probably account for the higher approach speeds in addition to the wind component factor.



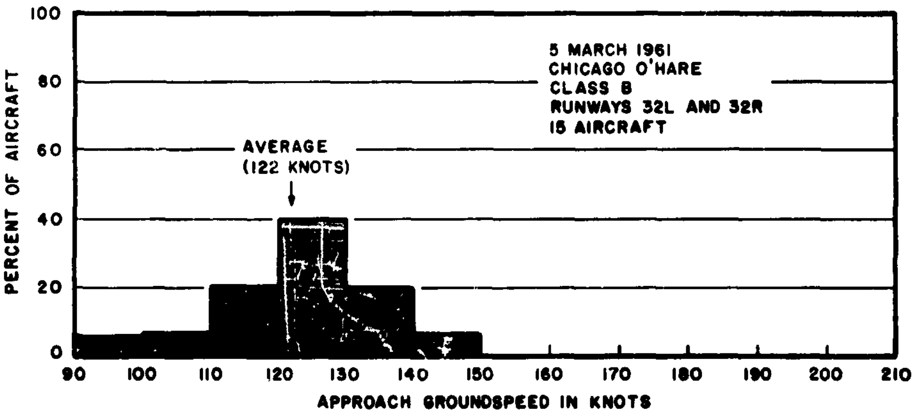
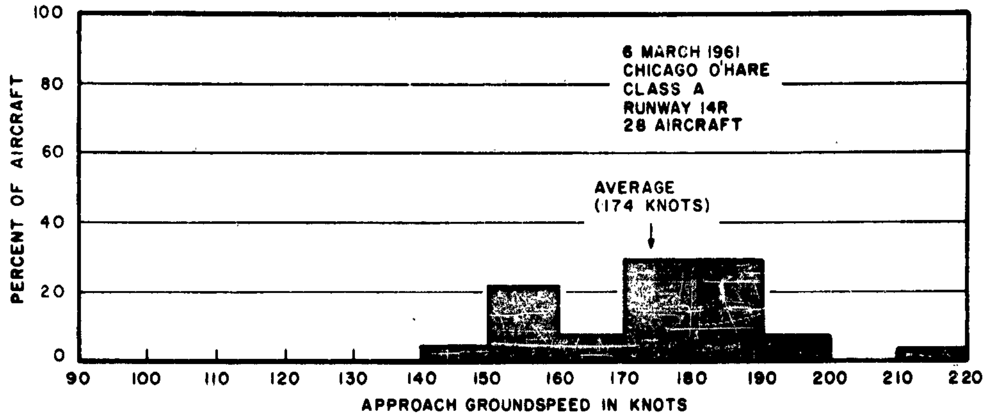
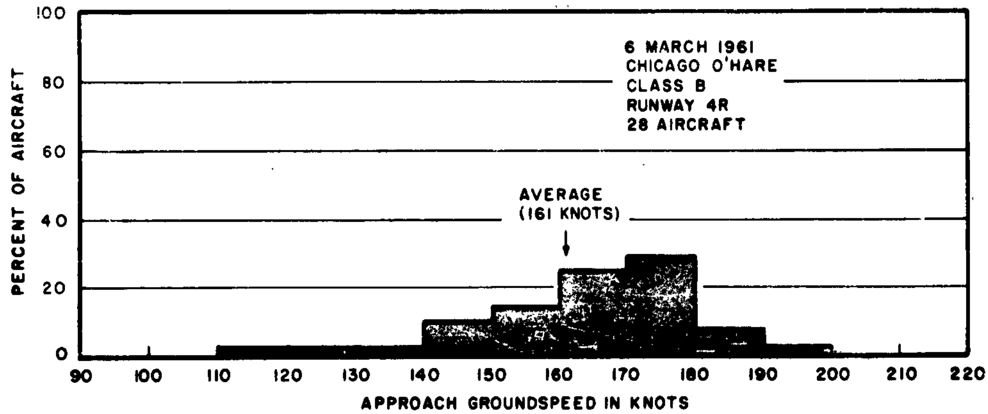


FIGURE D-1. AIRCRAFT APPROACH SPEEDS FROM 10 TO 5 MILES
SHEET 1 OF 4





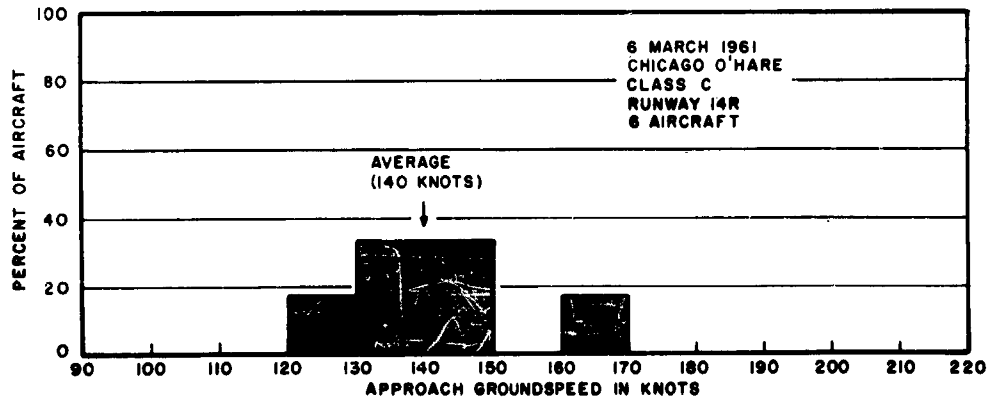
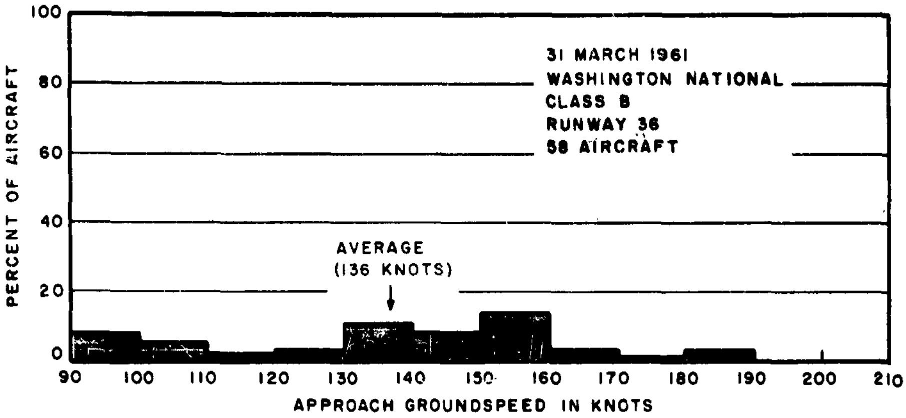
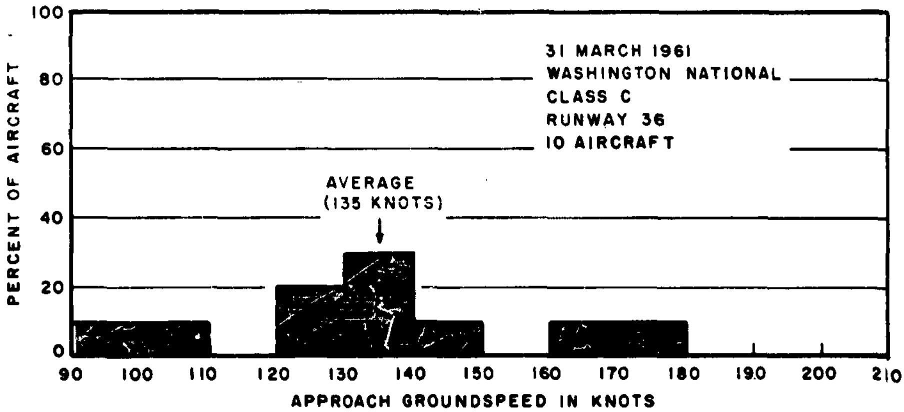


FIGURE D-1
SHEET 2 OF 4





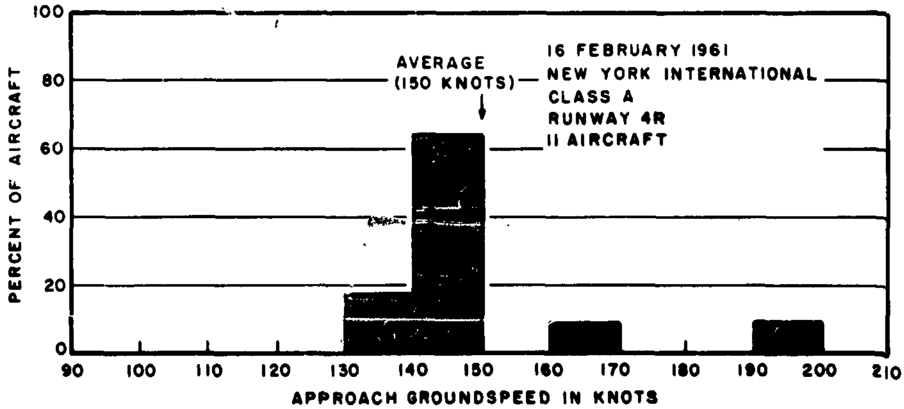
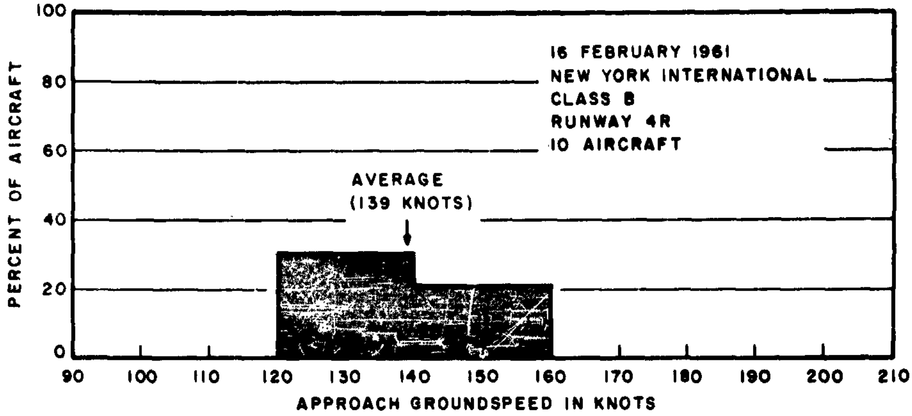
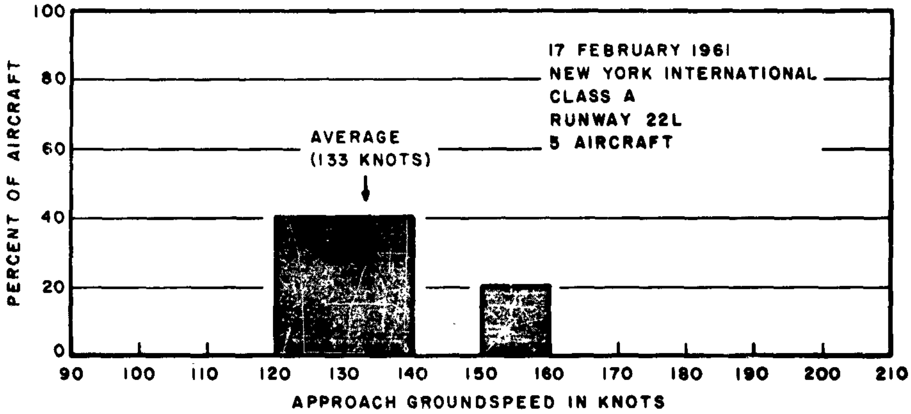


FIGURE D-1
SHEET 3 OF 4





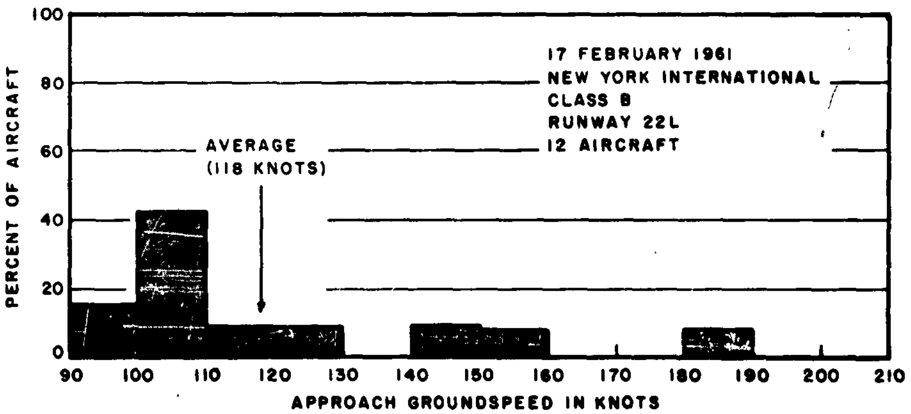


FIGURE D-1
SHEET 4 OF 4

APPENDIX E

MATHEMATICAL DESCRIPTION OF MULTI-SERVER QUEUING MODEL USED TO COMPUTE GATE DELAY

This appendix is concerned with the numerical treatment of a queuing problem that has constant service time and many servers. It is expected that this treatment will approximate the solution of a problem regarding the number of gates required to adequately serve the demand by aircraft. Since this demand fluctuates with respect to time, the desired solution is essentially time dependent not steady state.

The time axis ($t > 0$) is divided into equal increments of size Δt ($t_i = i\Delta t$). Service is constant and equal to $n\Delta t$. We define:

$a(k, i)$ = Probability of k arrivals (new customers) in time $(i - n)\,\Delta t < t \leq i\Delta t$

$A(k, i) = \sum_{j=k+1}^{\infty} a(j, i)$ = (cumulative) probability of more than k new customers in customers in time $(i - n)$ $\Delta t < t \leq i\Delta t$

$p(k, i)$ = Probability that there are k customers present (including those being serviced) at time $t = i\Delta t$.

$q(k, i) = \sum_{j=k+1}^{\infty} p(j, i)$ = (cumulative) probability of more

than k customers present (including those being serviced) at time $t = i\Delta t$.

The number of servers is m.

The logical treatment of this problem stems from the observation that the number of customers at time t_i is the number present at t_{i-n}, diminished at most by m (the number of servers) and increased by the number of new arrivals since t_{i-n}. Then,

$$p(0, i) = a(0, i) \sum_{k=0}^{m} p(k, i - n)$$

$$p(1, i) = a(1, i \sum_{k=0}^{m} p(k, i - n) + a(0, i) p(m + 1, i - n)$$

In general,

$$p(j, i) = a(j, i) \sum_{k=0}^{m} p(k, i - n) + a(j - 1, i) p (m + 1, i - n) + a(j - 2, i) p(m + 2, i - n) + \ldots + a(0, i) p(m + j, i - n) \qquad \text{(E-1)}$$

The cumulative form is given by

$$q(k, i) = A(k, i) + \sum_{j=0}^{K} a(j, i) q(m + k - j, i - n) \qquad \text{(E-2)}$$

In particular, it is assumed that the arrival distribution is Poisson. Let λ_i be the average number of new customers arriving in time $t_{i-1} < t \leq t_i$. Define

$$\overline{\lambda}_i = \sum_{j=i-n+1}^{i} \lambda_j$$

$$a(0, i) = e^{-\overline{\lambda}_i}$$

$$a(k, i) = \frac{\overline{\lambda}_i}{k} a(k - 1, i) \quad k \geq 1$$

Starting with the boundary conditions:

$$\lambda_i = 0 \quad i < 1$$

$$p(k, i) = 0 \quad i < 1 \text{ and all } k.$$

Equations E-1 and E-2 are computed successively to maximum time of interest. Equation E-2 then gives, for each time interval i, the probability that there are more than k customers waiting for gates and being serviced in the gates. Thus, the probability that all gates are full and no aircraft is waiting is obtained by k = m.

APPENDIX F

RUNWAY/TAXIWAY CROSSING

The previous report described the use of the Pre-emptive Poisson Arrivals Model (PAM) for computing delays at runway/taxiway intersections. The PAM model assumes a Poisson (random) input for both of two conflicting streams of aircraft, but one stream of aircraft has complete priority over the other. The latter is true of a runway/taxiway crossing where the aircraft on the runway (arrivals or departures) have priority over the taxiing aircraft. However, it has been determined that it is not correct to assume that both streams of traffic are Poisson.

In the case of arrivals on a runway, it has been shown that the original input of arrivals can be assumed to be Poisson, but that at the runway threshold the arrivals are necessarily spaced. In other words, the landings on a runway are the output of a queue. Thus, at the point where the landings and the taxiing aircraft conflict, the landing (arrival) traffic is spaced.

If we now consider departures (takeoffs) using a runway conflicting with taxiing aircraft, a similar situation exists in that takeoffs are spaced--being the output of a queue.

Therefore, if it can be established that the taxiing aircraft have a Poisson input the conditions are correct for application of SAM.

For the application of SAM to obtain delay (for taxiing aircraft) the following inputs apply:

1. <u>Runway Used for Takeoffs Only</u>

   λ_L now becomes λ_R where λ_R is the takeoff rate per hour (priority traffic)

λ_T - now becomes the hourly rate at which taxiing aircraft wish to cross the takeoff runway,

T - interval between successive taxiing aircraft,

F - time required to release one taxiing aircraft in between the takeoff sequence (from "clear to cross" to when aircraft is clear of the runway),

C - time from clear to takeoff to start roll for takeoffs,

R - runway time for takeoffs from start roll to passing through taxiway intersection.

Thus, if one runway has a number of taxiway intersections, SAM must be used individually for each intersection where λ_T is the rate for the particular intersection.

2. RUNWAY BEING USED FOR LANDINGS ONLY:

λ_R - the landing rate per hour,

λ_T - hourly rate at which taxiing aircraft wish to cross the landing runway,

T - interval between successive taxiing aircraft,

F - time required to release one taxiing aircraft in between the landing sequence,

C - commitment time for landings,

R - runway time for landings from over threshold to passing through taxiway intersection.

The only question now remaining is whether the taxiing aircraft have a Poisson input. If the taxiing aircraft are departures that have left the passenger terminal enroute to a different runway to takeoff there is good reason to believe that the input of such aircraft (λ_T) would be Poisson. Although departures may be scheduled for certain departure times, lateness in gate departure, devious taxiway routing, and the addition of general-aviation aircraft would undoubtedly randomize the flow.

If the taxiing aircraft are aircraft that have landed on a different runway, the situation might be slightly different. If the problem concerned two close parallel runways where aircraft that had landed on one runway had to cross the other runway, the input of taxiing aircraft might not be Poisson since the intervals between the landings are not Poisson.

However, if the runways were further apart with some taxiway interconnections prior to the crossing, differences in taxi speeds and path lengths might well randomize the demand of the taxiing aircraft.

Also, there is the case of a runway which is used for both takeoffs and landings. In this case, inputs 1 and 2 may be combined by using weighted averages of C and R. Thus,

- λ_R - the combined landing and takeoff rate per hour,
- λ_T - hourly rate at which taxiing aircraft wish to cross the landing/takeoff runway,
- T - interval between successive taxiing aircraft,
- F - time required to release one taxiing aircraft in between the landing/takeoff sequence.
- C - commitment time for landings (multiplied by the landing probability) plus, clear to take-off to start roll for takeoffs (multiplied by the takeoff probability).
- R - runway time for landings (multiplied by the landing probability) plus runway time for takeoffs (multiplied by the takeoff probability).

In conclusion, it may be stated that a special application of SAM will allow the computation of taxiway delay at taxiway/runway intersections for the following cases: (1) departures taxiing across a runway used for takeoffs only, landings only or both landings and takeoffs; and (2) arrivals taxiing across a runway used for takeoffs only, landings only or both landings and takeoffs provided that the arrival (taxiing) input is Poisson.